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ON SOME PROPERTIES OF BITOPOLOGICAL QHC SPACES

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Abstract. Some important questions connected with bitopological QHC spaces are investigated. New conditions are found, under which such spaces are compact with respect to one component of the topology. It is shown that a pairwise extremal disconnected bitopological QHC space is S-closed in the sense of [4]. Theorems on the second category of a base set and on the almost Baire property of bitopological QHC spaces are proved. Also, several properties of QHC bitopological spaces are found under some known bitopological mappings.

Keywords: bitopological QHC space, compactness, pairwise extremal disconnectedness, almost Baire bitopological space, pairwise continuous mapping.

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1. INTRODUCTION AND PRELIMINARIES

The basic concepts of bitopological spaces are introduced in [12] as a tool of investigating quasimetric spaces. According to [12], by a bitopological space (briefly bispace) we understand a triple (X, τ_1, τ_2), where topologies τ_1 and τ_2 are defined on a base set X. In asymmetric topology, bitopological spaces play a key role along with quasiproximity and quasiuniform spaces (see, e.g., [13]). The investigation of bitopological structures makes it possible to characterize a great number of asymmetric objects that have been left outside the framework of classical (symmetric) topology. Moreover, the bitopological methods of investigation are successfully used in other mathematical disciplines (see, e.g., [7], [11], [13], [14], etc.).

In this paper, we study some important properties of quasi H-closed (briefly, QHC) bispaces. Keeping in mind the fact that H-closed topological spaces are the object of numerous investigations in general topology and that vast fundamental knowledge has been accumulated in this direction, here we make an attempt to deepen the interest in studying such spaces. The notion of a QHC bispace was first introduced in [16], as a natural generalization of a H-closed topological space. Interesting results and sufficiently comprehensive characterizations of QHC spaces can be found in [6], [16], [17], etc.

In Section 2, we present some new structural properties of QHC spaces and also discuss their behavior under the known bitopological mappings.

Our main sources of reference are [2], [8] for topological spaces and [7] for bitopological ones.

Throughout the paper, we use the following notation: the interior and closure of a set $A \subset X$ with respect to the topology τ_i are denoted by $\tau_i intA$ and $\tau_i clA$, respectively, where $i \in \{1; 2\}$. If O is open in τ_i, then we write $O \in \tau_i$, while, for the τ_i-closed set F, we use the notation $F \in co\tau_i$ (in this case, for brevity, O and F are meant also as an i-open and an i-closed set, respectively). Denote by $\tau_i^* = \tau_i \cap A$ the topology induced on the set A from the τ_i. The family of all i-open neighborhoods of a subset M of the bispace (X, τ_1, τ_2) is denoted by $\sum_i^X(M)$, and the class of all i-dense subsets by $i - D(X)$. The family of all i-nowhere dense sets is denoted by $i - ND(X)$. We write $X \in i - CatgI$ (or $X \in i - CatgII$), when the basic set X of (X, τ_1, τ_2) is of the first (or second category).

The following families of sets play a special role below: $(i, j) - RO(X) = \{A|A = \tau_i int\tau_j clA\}$ and $(i, j) - RC(X) = \{A|A = \tau_i cl\tau_j intA\}$ denote the classes of (i, j)-regularly open and (i, j)-regularly closed sets of the bispace (X, τ_1, τ_2); $(i, j) - SO(X) = \{A|A \subset \tau_j cl\tau_i intA\}$ is the class of all (i, j)-semiopen sets, and the members of $i - Clp(X) = \{A|A \in \tau_i \cap co\tau_i\}$ are said to be clopen sets. In [5], we introduce and apply to the study several bitopological properties of the class of special-type sets: a set $A \subset X$ is said p-open if it can be represented as the intersection of two different sets $A_1 \in \tau_1\backslash\{\varnothing; X\}$ and $A_2 \in \tau_2\backslash\{\varnothing; X\}$. According to [17], a subset A of (X, τ_1, τ_2) satisfies the condition C_{ij} iff $\tau_i cl\tau_j \mathrm{int}A \subset \tau_j cl\tau_j \mathrm{int}A$. In our considerations, a restricted case of this property is presented. Naturally, if the property C_{ij} is possessed only by (i, j)-regular closed sets, then we say that (X, τ_1, τ_2) satisfies the $C_{ij}(RC)$-condition.

Next, in several results, we apply few important notions on bitopological structures, which are completely concerned in [7]. A bispace (X, τ_1, τ_2) is said to be (i, j)-regular if, for a point $x \in X$ and a set $x \notin F \in co\tau_i$, there are disjoint sets $U \in \sum_i^X(x)$ and $V \in \sum_j^X(F)$ [12]. It can be easily to verify that a bispace (X, τ_1, τ_2) is (i, j)-regular iff, for every point $x \in X$ and any $U \in \sum_i^X(x)$, there exists $V \in \sum_j^X(F)$ such that $\tau_j clV \subset U$. Usually, a bispace (X, τ_1, τ_2) is called to be (i, j)-semiregular (brief. $(i, j) - SR$) if, for every $G \in \tau_i$, there is $V \in (i, j) - RO(X)$ such that $V \subset G$. Suppose that, for any point $x \in X$ and every $G \in \sum_i^X(x)$, there exists $x \in V \in (i, j) - RO(X)$ such that $V \subset G$. Then we say that (X, τ_1, τ_2) is locally (i, j)-semiregular bispace (brief. $(i, j) - LSR$). (X, τ_1, τ_2) is called p-Urysohn (resp. almost or A-(i, j)-Urysohn) if, for every pair of different points $(x; y) \in X^2$, there exists neighborhoods $U \in \sum_i^X(x)$ and $V \in \sum_j^X(y)$ (resp. $V \in \sum_i^X(y)$) such that $\tau_j clU \cap \tau_i clV = \varnothing$ (resp. $\tau_j clU \cap \tau_j clV = \varnothing$). It is well known that the peculiarity of extremal disconnectedness makes important connections between several topological constructions . Bitopologically modified spaces of this topological notion have many interesting applications (see, e.g., [15], [7]). According [15], a bispace (X, τ_1, τ_2) is called to be p-extremally disconnected (brief. p-E.D.) if $\tau_j clO \in \tau_i$, for any $O \in \tau_i$. It can be easily to verify that (X, τ_1, τ_2) is p-E.D. iff $\tau_i clO_1 \cap \tau_j clO_2 = \varnothing$ for any pair of disjoint sets $O_1 \in \tau_j$ and $O_2 \in \tau_i$.

2. THE MAIN RESULTS

A bispace (X, τ_1, τ_2) is called (i, j)-QHC if any covering $V = \{O_\alpha \in \tau_i\}_{\alpha\in\Lambda}$ of X contains a finite subfamily $V' = \{O_{\alpha_k} \in V\}_{k=\overline{1;n}}$ such that $X = \cup_{k=1}^{n} \tau_j cl O_{\alpha_k}$ [16], [17]. It is known that (X, τ_1, τ_2) is (i, j)-QHC iff any centered system $\{U_\alpha \in \tau_i\}_{\alpha\in\Lambda}$ satisfies the condition $\cap_{\alpha\in\Lambda} \tau_j cl U_\alpha \neq \varnothing$ [16]. It is obvious that every i-compact bispace is an (i, j)-QHC but the opposite does not hold in general.

Before formulating our first result, we take into account two important known theorems on QHC bispaces. One of these theorems establishes a connection between compactness and QHC.

THEOREM 2.1 [16]. *Let a bispace (X, τ_1, τ_2) be (i, j)-QHC and (i, j)-regular. Then it is an i-compact.*

The other, not less interesting, theorem gives conditions under which a subspace of the QHC-bispace is QHC.

THEOREM 2.2 [17]. *Let an (i, j)-QHC bispace (X, τ_1, τ_2) satisfy the $C_{ij}(RC)$-property. Then, for any $A \in (i, j) - RC(X)$, a subspace (A, τ_1^*, τ_2^*) is an (i, j)-QHC.*

Based on the above-mentioned theorems and using the constructions from [2] to investigate H-closed topological spaces, we have established new conditions for QHC bispaces to be compact.

THEOREM 2.3. *Let an (i, j)-QHC bispace (X, τ_1, τ_2) be $(i, j) - SR$, $A - (i, j)$-Urysohn, and having the $C_{ij}(RC)$-property. Then it is i-compact space.*

Proof. We first must show that (X, τ_1, τ_2) is (i, j)-regular. To this purpose, consider a point $x_0 \in X$ and its arbitrary neighborhood $O(x_0) \in \sum_i^X(x_0)$. Then from the $(i, j) - SR$ property of (X, τ_1, τ_2) it follows $O'(x_0) \in (i, j) - RO(X)$ such that $O'(x_0) \subset O(x_0)$. Since $M \equiv X\backslash O'(x_0) \in (i, j) - RC(X)$, Theorem 2.2 yields that (M, τ_1^*, τ_2^*) is an (i, j)-QHC subspace. Now for each point $x \in M$ and $x_0 \in X$ consider sets $U(x) \in \sum_i^X(x)$ and $U_x(x_0) \in \sum_i^X(x_0)$ such that $\tau_j clU(x) \cap \tau_j clU_x(x_0) = \varnothing$. Obviously, from the covering $\mathcal{F} = \{U(x)\}_{x\in M}$ we can choose a finite subfamily $\mathcal{F}_0 = \{U(x_k)\}_{k=\overline{1;n}} \subset \mathcal{F}$ such that $M \subset \cup_{k=1}^{n} \tau_j clU(x_k)$. By taking into consideration the neighborhoods $\{U_{x_k}(x_0)\}_{k=\overline{1;n}}$ of the point $x_0 \in X$, we note that $\tau_j clU_{x_k}(x_0) \cap \tau_j clU(x_k) = \varnothing$ or, equivalently, $\tau_j clU_{x_k}(x_0) \subset X\backslash\tau_j clU(x_k)$ for every $k = \overline{1;n}$. Let us suppose that $O''(x_0) \equiv \cap_{k=1}^{n} U_{x_k}(x_0) \in \tau_i$; then $x_0 \in O''(x_0) \subset \tau_j cl O''(x_0)$. Since the inclusion $\tau_j clU_{x_k}(x_0) \subset X\backslash\tau_j clU(x_k)$ holds for $\forall k = \overline{1;n}$, we have that $\cap_{k=1}^{n} \tau_j clU_{x_k}(x_0) \subset \cap_{k=1}^{n}(X\backslash\tau_j clU(x_k))$. Hence, we have $\tau_j cl O''(x_0) = \tau_j cl(\cap_{k=1}^{n} U_{x_k}(x_0)) \subset \cap_{k=1}^{n} \tau_j clU_{x_k}(x_0) \subset \cap_{k=1}^{n}(X\backslash\tau_j clU(x_k)) \subset X\backslash M = O'(x_0) \subset O(x_0)$, and, consequently, the (i, j)-regularity of (X, τ_1, τ_2) is established. Finally, by Theorem 2.1 the compactness of the space (X, τ_i) follows.

It is known that a space (X, τ) is submaximal iff $D(X) \subset \tau\backslash\varnothing$ (some important properties of such spaces are given, for example, in [3]). Using this topological notion of submaximality, we claim the following theorem on second category.

THEOREM 2.4. *Let a bispace (X, τ_1, τ_2) be the (i, j)-QHC and i-submaximal. Suppose that, for every set $O \in \tau_i \backslash \varnothing$, there exists $U \in \tau_i \backslash \varnothing$ such that $\tau_j clU \subset O$. Then the set $X \in i - CatgII$.*

Proof. Suppose that $X \in i - CatgI$, i.e., $X = \cup_{n=1}^{\infty} A_n$, where $A_n \in i - ND(X)$ for every $n \in \mathbb{N}$. From the implication $A_n \in i - ND(X)$ it follows that $D_n \equiv (X \backslash A_n) \in i - D(X)$ and by the submaximality of (X, τ_i) we have $D_n \in \tau_i$. The conditions directly imply the existence of a set $U_1 \in \tau_i \backslash \varnothing$ such that $\tau_j clU_1 \subset D_1$ with $U_1 \cap D_2 \in \tau_i \backslash \varnothing$. Consequently, for the set $U_1 \cap D_2 \in \tau_i \backslash \varnothing$, there is $U_2 \in \tau_i \backslash \varnothing$ such that $\tau_j clU_2 \subset U_1 \cap D_2 \subset D_2$. Continuing this process, we obtain a sequence of sets $\{U_n | U_n \in \tau_i \backslash \varnothing\}_{n \in \mathbb{N}}$ such that $U_1 \supset U_2 \supset \cdots$, where $\tau_j clU_n \subset D_n$ for any $n \in \mathbb{N}$. It is obvious that $\{U_n\}_{n \in \mathbb{N}}$ is a centered family of i-open sets. Since (X, τ_1, τ_2) is an (i, j)-QHC bispace, we have that $\cap_{n=1}^{\infty} \tau_j clU_n \neq \varnothing$. Therefore, $\cap_{n=1}^{\infty} D_n \neq \varnothing$, a contradiction to our assumption.

Now we will establish another interesting connection between (i, j)-QHC and almost (i, j)-Baire spaces (brief. $A - (i, j)$-Baire space). Some characterizations of the Baire property in the bitopological case can be found in [1], [5], [7] and [10]. To avoid confusion, we use the definition of the almost Baire property from [7]. The bispace (X, τ_1, τ_2) is said to be $A - (i, j)$-Baire, iff any sequence of sets $\mathcal{F} = \{G_n | G_n \in \tau_j \cap i - D(X)\}_{n \in \mathbb{N}}$ satisfies the condition $\cap_{n \in \mathbb{N}} G_n \in i - D(X)$.

THEOREM 2.5. *Let a bispace (X, τ_1, τ_2) be (i, j)-QHC. Suppose that, for every p-open set $O \subset X$, there exists $G \in \tau_i$ such that $\tau_j clG \subset O$. Then (X, τ_1, τ_2) is $A - (i, j)$-Baire bispace.*

Proof. Consider any family $\{U_n \in \tau_j \cap i - D(X)\}_{n \in \mathbb{N}}$ and define by induction a sequence $\{V_n \in \tau_i\}_{n \in \mathbb{N}}$ such that $\tau_j clV_n \subset U_n$ for $\forall n \in \mathbb{N}$ with $V_1 \supset V_2 \supset \cdots$. Suppose that $W \in \tau_i$ is an arbitrary nonempty set. Since $U_n \in i - D(X)$, there exists $V_1 \in \tau_i$ such that $\tau_j clV_1 \subset W \cap U_1 \neq \varnothing$. Consequently, V_1 is constructed. The second step of the induction requires the assumption that the sets V_n are well-defined, just as above, for all $n \leq k$. Therefore, we take as V_{k+1} an arbitrary set $V_{k+1} \in \tau_i$ such that $\tau_j clV_{k+1} \subset V_k \cap U_{k+1} \neq \varnothing$. By such an algorithm the sets V_n, $n \in \mathbb{N}$, can be also constructed in a similar way. It can be easily seen that $W \cap \left(\cap_{n=1}^{\infty} U_n\right) \supset \cap_{n=1}^{\infty} \tau_j clV_n$; moreover, the family $\{V_n \in \tau_i\}_{n \in \mathbb{N}}$ represents the centered system in (X, τ_1, τ_2). Since (X, τ_1, τ_2) is (i, j)-QHC, we have $\cap_{n=1}^{\infty} \tau_j clV_n \neq \varnothing$, i.e., $W \cap \left(\cap_{n=1}^{\infty} U_n\right) \neq \varnothing$ for any $W \in \tau_i$, and this yields that $\cap_{n=1}^{\infty} U_n \in i - D(X)$.

The notion of an $(i, j) - S$-closed space is given in [4], where some of its properties are also studied. In the definition of QHC bispace replacing an i-open covering by an (i, j)-semiopen covering, we come to the notion of an $(i, j) - S$ closed space. Below we show how $(i, j) - S$ closed and (i, j)-QHC spaces are related to each other.

THEOREM 2.6. *If a bispace (X, τ_1, τ_2) is both (i, j)-QHC and $p - E.D.$, then it is $(i, j) - S$ closed.*

Proof. Let us consider an arbitrary covering $\{U_\alpha | U_\alpha \in (i,j) - SO(X)\}_{\alpha\in\Lambda}$ of X. Then, for every $U_\alpha \in (i,j) - SO(X)$, the family $W = \{W_\alpha = \tau_j cl \tau_i int U_\alpha\}_{\alpha\in\Lambda}$ also covers X. From the $p - E.D.$ of (X, τ_1, τ_2) it follows that $W_\alpha \in \tau_i$ for every $\alpha \in \Lambda$. Since (X, τ_1, τ_2) is (i,j)-QHC, from the cover W one can choose some finite subfamily $\{W_{\alpha_k} \in W\}_{k=\overline{1;n}}$ such that $X = \cup_{k=1}^n \tau_j cl W_{\alpha_k}$, i.e., $X = \cup_{k=1}^n \tau_j cl \tau_i int U_{\alpha_k} = \cup_{k=1}^n \tau_j cl U_{\alpha_k}$.

To continue, we will investigate, from the standpoint of dynamics, the behavior of QHC spaces under various known bitopological mappings. According to [12], a map $f\colon (X, \tau_1, \tau_2) \to (Y, \gamma_1, \gamma_2)$ is said to be p-continuous if both $f\colon (X, \tau_1) \to (Y, \gamma_1)$ and $f^{-1}\colon (X, \tau_2) \to (Y, \gamma_2)$ are continuous maps.

THEOREM 2.7. *Let (X, τ_1, τ_2) be an (i,j)-QHC bispace and a map $f\colon (X, \tau_1, \tau_2) \to (Y, \gamma_1, \gamma_2)$ be a p-continuous surjection. Then (Y, γ_1, γ_2) is a (i,j)-QHC bispace.*

Proof. Consider any covering $U = \{O_\alpha | O_\alpha \in \gamma_i\}_{\alpha\in\Lambda}$ of (Y, γ_1, γ_2). Then, obviously, the family of i-open sets $W = \{f^{-1}(O_\alpha) | O_\alpha \in U\}_{\alpha\in\Lambda}$ covers the space (X, τ_1, τ_2). Therefore, the covering W contains some finite subfamily $\{f^{-1}(O_{\alpha_k})\}_{k=\overline{1;n}}$ such that $X = \cup_{k=1}^n \tau_j cl f^{-1}(O_{\alpha_k})$. By the j-continuity of the map f yields that $Y = f(\cup_{k=1}^n \tau_j cl f^{-1}(O_{\alpha_k})) = f(\tau_j cl f^{-1}(\cup_{k=1}^n O_{\alpha_k})) \subset \cup_{k=1}^n \gamma_j cl O_{\alpha_k}$, i.e., the bispace (Y, γ_1, γ_2) is (i,j)-QHC.

In Theorem 2.8, we use the following fact: If, in an (i,j)-regular bispace (X, τ_1, τ_2), a set $A \in j - Clp(X)$, then (A, τ_1^*, τ_2^*) is (i,j)-regular, as one can easily verify. Moreover, for bitopological mappings,we need the following useful notion. Let $f\colon (X, \tau_1, \tau_2) \to (Y, \gamma_1, \gamma_2)$ be a p-continuous bijective action together with p-continuous $f^{-1}\colon (Y, \gamma_1, \gamma_2) \to (X, \tau_1, \tau_2)$. Then f is called p-homeomorphism. Here we give conditions under which the j-homeomorphy of f implies its p-homeomorphy.

THEOREM 2.8. *Consider bispaces (X, τ_1, τ_2) and (Y, γ_1, γ_2), where (X, τ_1, τ_2) is $(i,j) - LSR$ and (i,j)-QHC with the $C_{ij}(RC)$-property and (Y, γ_1, γ_2) is i-Hausdorff and (i,j)-regular. Let a j-homeomorphism $f\colon (X, \tau_1, \tau_2) \to (Y, \gamma_1, \gamma_2)$ be i-continuous, and let $f(A) \in j - Clp(Y)$ for every $A \in (i,j) - RC(X)$. Then the map f is a p-homeomorphism.*

Proof. Our main purpose is to show the i-openness of the map f, i.e., that $f(O) \in \gamma_i$ for every $O \in \tau_i$. If we fix an arbitrary set $O \in \tau_i$ and a point $y \in f(O)$, then there exists a point $x \in O$ such that $y = f(x)$. The $(i,j) - LSR$ property of (X, τ_1, τ_2) implies the existence of a set $x \in V \in (i,j) - RO(X)$ such that $V \subset O$. Moreover, $y \in f(V) \subset f(O)$. Now we shall show that $f(U) \in \gamma_i$ for every $U \in (i,j) - RO(X)$. By Theorem 2.2 this implies that the induced subspace (G, τ_1^*, τ_2^*) is an (i,j)-QHC, where $G \equiv X \backslash U \in (i,j) - RC(X)$. Since $M \equiv f(G) \in j - Clp(Y)$, this directly yields that $(M, \gamma_1^*, \gamma_2^*)$ is (i,j)-QHC. Moreover, we have that $(M, \gamma_1^*, \gamma_2^*)$

is an (i, j)-regular bispace. Therefore, Theorem 2.1 implies that (M, γ_i^*) is an i-compact subspace in the Hausdorff space (Y, γ_i). Therefore, $M \in co\gamma_i$, or, equivalently, $f(X \backslash U) \in co\gamma_i$ and $f(U) = Y \backslash f(X \backslash U) \in \gamma_i$, respectively. Obviously, the implications $y \in f(V) \in \gamma_i$ and $f(O) \in \gamma_i$ are valid.

A map $f\colon (X, \tau_1, \tau_2) \to (Y, \gamma_1, \gamma_2)$ is called $(i, j) - \theta$ continuous if for any point $x_0 \in X$ and every neighborhood $U \in \sum_i^Y(y_0)$ of the point $y_0 = f(x_0)$, there exists $V \in \sum_i^X(x_0)$ such that $f(\tau_j clV) \subset \gamma_j clU$ (see, e.g., [7]).

As in [5], we call a set K to be p-dense in a bispace (X, τ_1, τ_2) if, for any point $x \in X$ and its p-open neighborhood $W(x)$ we have $W(x) \cap K \neq \varnothing$ (brief. $K \in p$-D(X)).

PROPOSITION 2.1. *Let a map $f\colon (X, \tau_1, \tau_2) \to (Y, \gamma_1, \gamma_2)$ satisfy the condition $f(X) \in p - D(Y)$, and let the maps $g, h\colon (Y, \gamma_1, \gamma_2) \to (Z, \omega_1, \omega_2)$ be $(i, j) - \theta$ and $(j, i) - \theta$ continuous, respectively. If (Z, ω_1, ω_2) is a p-Urysohn bispace, then $g \circ f = h \circ f$ implies $g = h$.*

Proof. Suppose that $g(y) \neq h(y)$ for some $y \in Y$. Then there are sets $U \in \sum_i^Z(g(y))$ and $V \in \sum_j^Z(h(y))$ such that $\omega_j clU \cap \omega_i clV = \varnothing$. Since g and h are $(i, j) - \theta$ and $(j, i) - \theta$ continuous maps, respectively, then there are $U' \in \sum_i^Y(y)$ and $V' \in \sum_j^Y(y)$ such that the inclusions $g(\gamma_j clU') \subset \omega_j clU$ and $h(\gamma_i clV') \subset \omega_i clV$ are valid. It is clear that $y \in W \equiv U' \cap V'$ and that W is a p-open set. Note that $g(\gamma_j clW) \subset \omega_j clU$ and $h(\gamma_i clW) \subset \omega_i clV$. Since $f(X) \in p - D(Y)$, we have that $W \cap f(X) \neq \varnothing$, i.e., there exists a point $x \in X$ such that $f(x) \in W$. By the inclusions $g(W) \subset \omega_j cl\ U$ and $h(W) \subset \omega_i clV$, it follows that $g(f(x)) \in \omega_j clU$ and $h(f(x)) \in \omega_i clV$. Therefore, from $\omega_j clU \cap \omega_i clV = \varnothing$ it follows that $(g \circ f)(x) \neq (h \circ f)(x)$. This contradiction shows that $f = g$.

Finally, from bitopological point of view, we present a version of a result of Fomin [9].

THEOREM 2.9. *Let (X, τ_1, τ_2) be an (i, j)-QHC bispace, and let $f\colon (X,\tau_1,\tau_2) \to (Y, \gamma_1, \gamma_2)$ be an $(i, j) - \theta$ continuous surjection. Then (Y, γ_1, γ_2) is an (i, j)-QHC bispace.*

Proof. Let $\{O_\alpha \in \gamma_i\}_{\alpha \in \Lambda}$ be some cover of (Y, γ_1, γ_2). Then, for every point $x \in X$, there exists an index $\alpha(x) \in \Lambda$ such that $f(x) \in O_{\alpha(x)}$. By the $(i, j) - \theta$ continuity of the map f, there exists $V(x) \in \sum_i^X(x)$ such that $f(\tau_j clV(x)) \subset \gamma_j clO_{\alpha(x)}$. It is clear that from an i-open cover $\{V(x)\}_{x \in X}$ of (X, τ_1, τ_2) one can choose a finite subfamily $\{V(x_k)\}_{k=\overline{1;n}} \subset \{V(x)\}_{x \in X}$ such that $X = \cup_{k=1}^n \tau_j clV(x_k)$. Since $Y = f(X) = \cup_{k=1}^n f(\tau_j clV(x_k)) \subset \cup_{k=1}^n \gamma_j clO_{\alpha(x_k)}$ we get that (Y, γ_1, γ_2) is an (i, j)-QHC.

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REZIUMĖ

I. Dochviri. Kai kurios bitopologinių QHC ***erdvių savybės***

Išnagrinėta keletas svarbių bitopologinių QHC erdvių savybių. Pavyzdžiui, apie tų erdvių bazinių aibių antrąją kategoriją, apie beveik Bero savybę ir pan.