



# Comparison of Mathematical Models of Torque Transmitted by Multi-disc Wet Clutch with Experimental Results

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**Abstract.** In the paper results of experimental tests on a multi-disc wet clutch are presented. Typical, single-sided multi-disc wet clutch was analysed. Experimentally obtained data present torque capacity of the clutch with varying number of friction surfaces. The results are also compared to characteristics obtained based on several known mathematical models. It was found that differences between most of known models describing torque transmitted by clutches and experimental results are considerable. The best correspondence between model and experiment was reached for Bąk's model. Nevertheless, differences between the model and data gained via experiment might differ substantially i.e., characteristic of proposed model are mostly shifted upwards in relation to experimental characteristics. The paper also includes a brief description of a test stand and its scheme. The article presents also a figure which contains diagrams presenting all characteristics as functions of average contact pressure on friction surfaces.

**Keywords:** Multi-disc clutch · Friction · Mathematical models · Experiments

## 1 Introduction

As a result of observable growth of application of semi and full powershift transmissions in heavy machinery like tractors, military vehicles or telescopic handlers, it is critical to gain knowledge about effects occurring in a wet clutch [1, 2]. It is especially important due to a number of clutches applied in those types of transmissions and their critical role in power transmission. Powershift transmission might have even more than ten wet clutches [2]. Therefore, a properly designed clutch with its dimensions and number of discs matching requirements is important. It might enhance design of assembly that correctly fulfils vital requirements such as sufficient torque capacity, minimalized number of discs of a clutch. Moreover, it is also required that slippage of the clutch will not occur. Unwanted slippage of engaged clutch would reduce efficiency of a vehicle. It would be extremely undesirable in relation to current trend, because many scientists concentrate on enhancing efficiencies through novel devices or improvements of already existing devices [2–6].

Multi-disc wet clutch is a clutch with multiple friction surfaces. It transmits torque due to friction occurrence on contacting discs. Increased number of friction surfaces, which is equivalent to increased number of discs comprising a clutch, allows to transmit higher torque compared to a clutch with only one friction surface. Typically applicable multi-disc is shown in the Fig. 1. Clamping force exerted on friction and separator discs causes engagement of the clutch. Therefore, torque is transmitted between two elements (input and output shafts) due to aforementioned friction. Disadvantages of multi-disc clutches relate to deformation of friction and separator discs, as well as blocking and pressure plates. Those effects might lead to non-uniform pressure distribution on contact surfaces [7–9].

In this paper, selected known mathematical models describing multi-disc clutches are presented. Characteristics obtained from those models are compared to experimental results. The results are presented for a clutch with defined dimensions, for a varying number of friction surfaces.

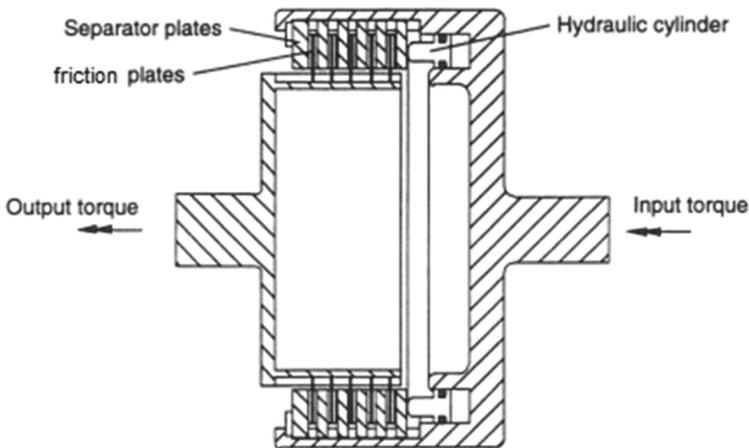
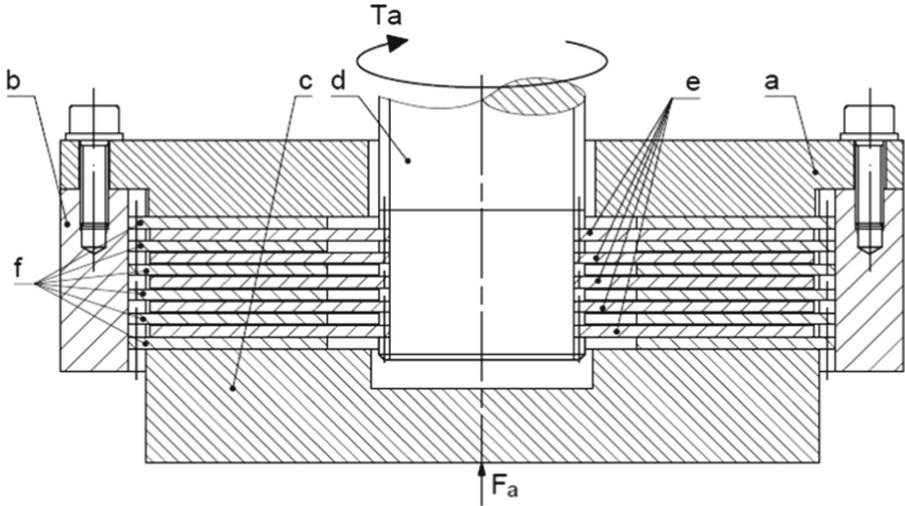


Fig. 1. Multi-disc wet clutch [10].

## 2 Mathematical Models

A typical model of a multi-disc clutch with a single-sided clamping of the plate pack is shown in Fig. 2. Friction discs with an internal spline  $e$  are connected to the shaft  $d$ , while discs  $f$  with an external spline are connected to the hub  $b$ . The clutch transmits torque to the hub from the shaft or in reversed direction, depending on the direction of transmission. This package is clamped by the axial force  $F_a$  exerted by the pressure plate  $c$  on the first plate. According to the model shown, the first disc is the separator disc. The blocking plate  $a$  restricts movement of clutch package in axial direction.



**Fig. 2.** Simplified sectional view of clutch assembly: a – blocking plate, b – hub, c – pressure plate, d – shaft, e – friction discs, f – separator discs,  $F_a$  – axial force,  $T_a$  – torque transmitted by a clutch [11].

In an engaged multi-disc clutch alternately mounted discs rotate at the same angular velocities. Therefore, in the models presented in this section, viscous friction is ignored [12]. In this case, the torque to be transmitted can be determined from widely known formula [10]:

$$T_a^{n_{pow}} = n \cdot T_t, \quad (1)$$

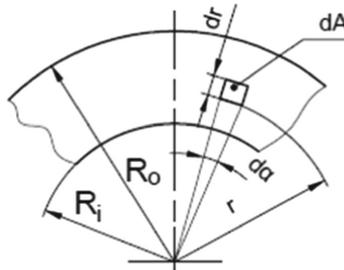
where  $n$  and  $T_t$  refer to number of friction surfaces and theoretical torque transmitted by single friction surface while friction forces appearing on splines are neglected. Theoretical torque  $T_t$  might be calculated with following equation:

$$T_t = \mu \cdot r_m \cdot F_a, \quad (2)$$

where  $\mu$  is friction coefficient for a pair of clutch discs and  $r_m$  refers to a mean radius defined as:

$$r_m = \frac{2 R_o^3 - R_i^3}{3 R_o^2 - R_i^2}. \quad (3)$$

Radius  $R_o$  and  $R_i$  are illustrated in Fig. 3.



**Fig. 3.** Dimensions of friction surface [13, 14].

Formula (1) does not take into account friction in spline connections. These forces are considered in a model developed by Osinski [15]:

$$T_{os} = k_{os} \cdot n \cdot T_t, \tag{4}$$

where  $k_{os}$  is a coefficient defining torque reduction depending on the number of friction surfaces  $n$  of the clutch. Values of the  $k_{os}$  coefficient are shown in Table 1.

**Table 1.** Values of coefficient  $k_{os}$  [15].

Number of friction surfaces	2	3	4	5	6	7	8	9	10
Coefficient $k_{os}$	1	0.97	0.94	0.91	0.88	0.85	0.82	0.79	0.76

Unfortunately, model proposed by Osinski applies arbitrarily assumed values of  $k_{os}$  coefficient. Influence of spline types or their standards is not taken into consideration in this model. Therefore, application of Eq. (4) might cause significant errors and faulty performances of wet clutches.

A mathematical model proposed by Båk describing torque transmitted by multi-disc clutch is shown below (Eq. (5)) [14]. According to the model, torque  $T_t^a$  is a sum of elementary torque transferred by each of  $i_{pc}$  friction surfaces.

$$T_t^a = F_a \cdot C \cdot \frac{d_{po}}{2} \cdot \cos\alpha_o \cdot \sum_{i_{pc}=1}^n \left[ \frac{\left( \frac{d_{pin}}{2} \cdot \cos\alpha_{in} - C \cdot \mu_{in} \right)^{\frac{i_{pc}}{2}}}{\left( \frac{d_{pin}}{2} \cdot \cos\alpha_{in} + C \cdot \mu_{in} \right)^{\frac{i_{pc}}{2}}} \cdot \frac{\left( \frac{d_{po}}{2} \cdot \cos\alpha_o - C \cdot \mu_o \right)^{\frac{i_{pc}-1}{2}}}{\left( \frac{d_{po}}{2} \cdot \cos\alpha_o + C \cdot \mu_o \right)^{\frac{i_{pc}-1}{2} + 1}} \right], \tag{5}$$

where  $\alpha$ ,  $d_p$  refer to pressure angle and pitch diameter of a spline, respectively. Subscripts  $o$  and  $in$  relate to external and internal splines. The same applies to  $\mu_o$  and  $\mu_{in}$ , which describe friction coefficient appearing on external and internal splines. Constant  $C$  is described by equation:

$$C = r_m \cdot \mu. \tag{6}$$

It is common that average contact pressure is applied in Eqs. (2) and (4) instead of axial force  $F_a$ . Average contact pressure  $p$  appearing on a friction surface might be

calculated with expression:

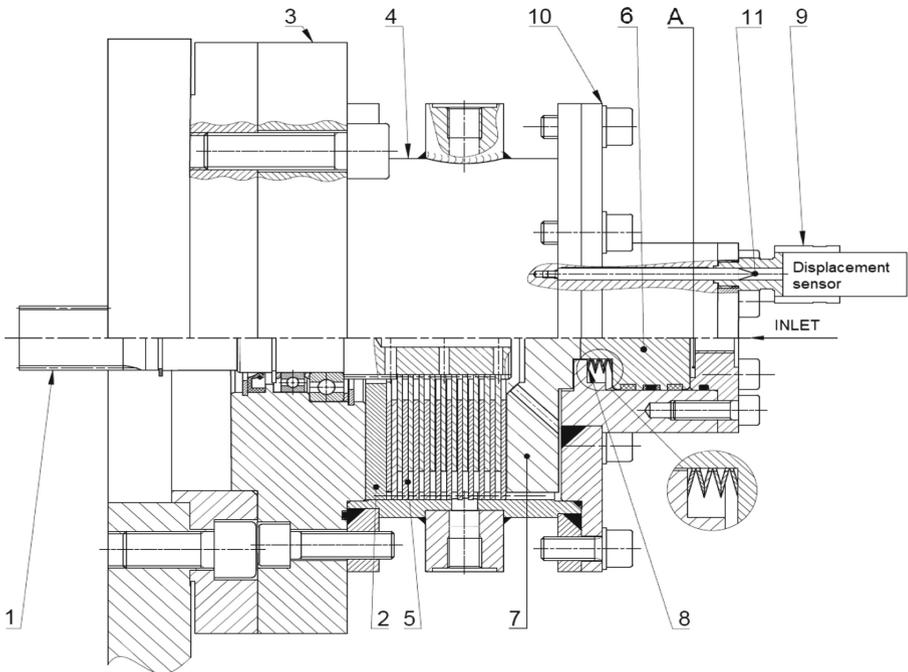
$$p = \frac{F_a}{\pi(R_o^2 - R_i^2)}. \quad (7)$$

### 3 Experiments

#### 3.1 Test Stand

In order to carry out research on multi-disc clutches a test stand was designed. The test stand gives opportunity to study torque capacity and durability of the clutch, hence versatility of the device is its main advantage. It allows also to conduct other tests of clutches, such as time of an engagement and disengagement or drag torque.

Test stand is presented in the Fig. 4. The torque transmitted by the clutch is measured by torque transducer. The torque transducer also connects hydraulic motor shaft with the apparatus shaft. The device during torque capacity tests works as a multi-disc brake. During those tests the hub is stationary, connected to test stand foundation, while the shaft rotates with friction discs. Separator discs are mounted in the hub, alternately with friction discs.

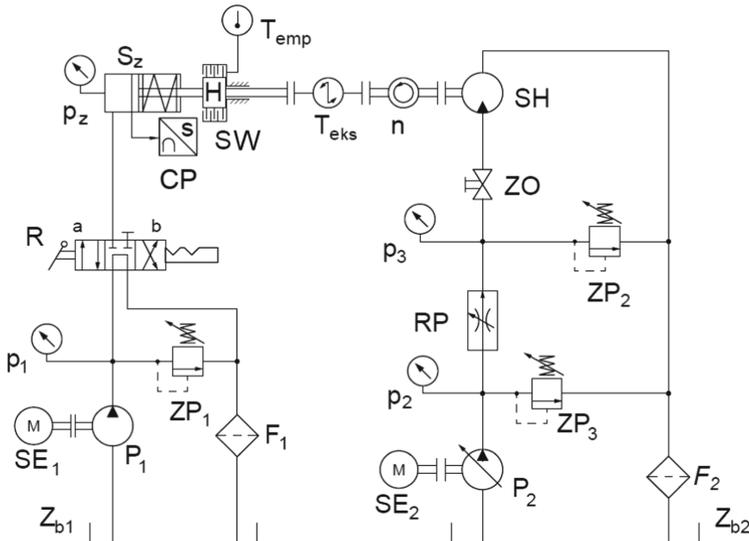


**Fig. 4.** Test stand: 1 – shaft, 2 – blocking plate, 3 – cover, 4 – hub, housing, 5 – clutch plates, 6 – piston, 7 – pressure plate, 8 – package of disc springs, 9 – displacement sensor holder, 10 – cover, 11 – position sensor tappet, A – actuator chamber [14].

As previously stated, the test stand allows conducting experiments for different combinations of number and dimension of friction and separator discs. Torque capacity of the clutch depends on axial force generated by the hydraulic actuator and the clutch itself. The axial force depends on pressure in the actuator chamber A. Axial force is applied by the piston to the pressure plate. Occurrence of axial force causes clamping of the clutch discs, hence engagement of the clutch.

Hydraulic circuit that supplies hydraulic actuator and motor is shown in the Fig. 5. For research on torque capacity tests a procedure was established. First, pressure  $p_1$  was adjusted by valve ZP<sub>1</sub>. Then, set-up of ZP<sub>3</sub> valve was slowly increasing, up to the moment when slippage occurred, which meant that torque capacity of the clutch had been exceeded. After the described cycle, the process continued further but with increased adjustment of ZP<sub>1</sub> valve. Between subsequent cycles force was not exerted on the pressure plate and shaft of hydraulic motor rotated at low rotational velocity for at least several seconds. An aim of the actions was to provide lubrication of every part of friction surfaces.

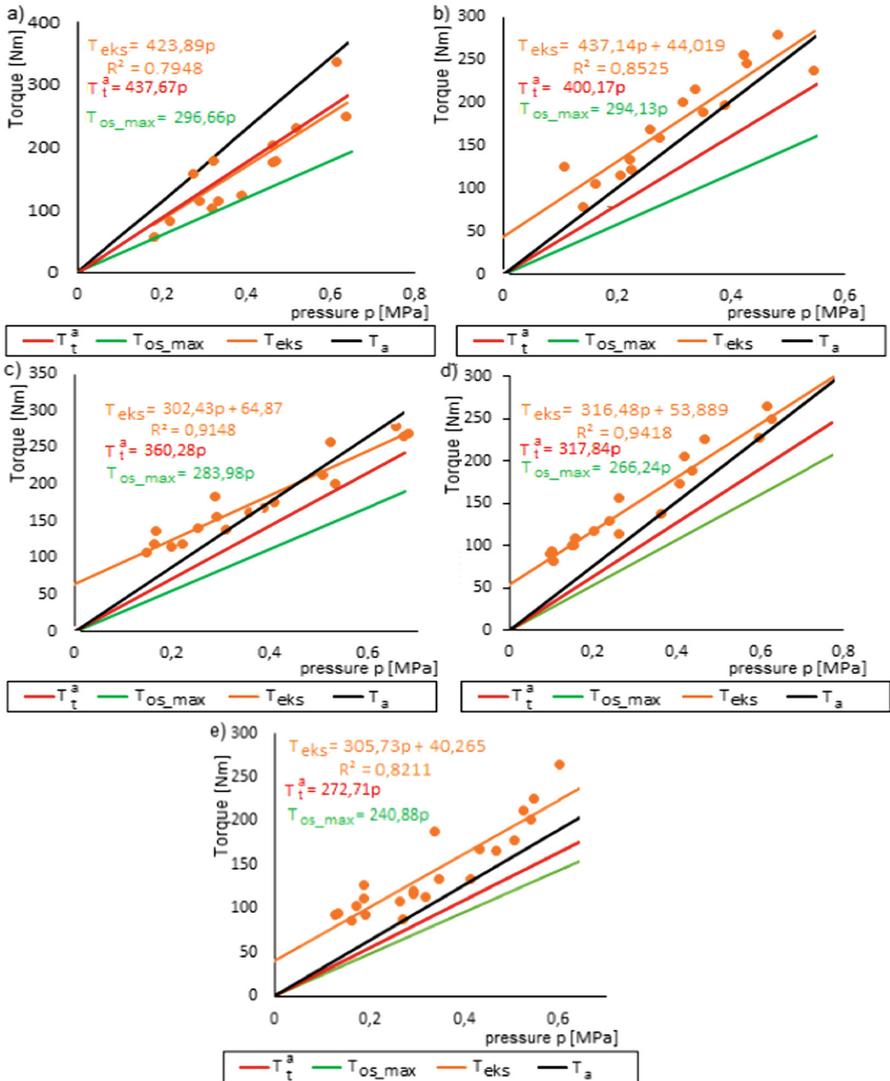
Axial force applied to the discs package is determined by pressure in actuator  $S_z$  chamber (A in Fig. 4) and varies as a function of pressure  $p_z$  adjusted by relief valve ZP<sub>1</sub>. Rotational velocity of hydraulic motor SH is set by two-port adjustable flow control valve RP. The valve set-up was adjusted to ensure rotational speed  $n$  of motor during disengagement of the clutch that does not exceed a hundred rpm. Maximum torque appearing on hydraulic motor shaft depends on adjustment of pressure  $p_3$ . All the signals from the sensors were recorded by Hydac HMG 4000.



**Fig. 5.** Hydraulic system supplying actuator and motor: P – pumps, ZP – relief valve, ZO – ball valve, RP – flow control valve, p – pressure transducers, R – 4/3 directional valve, CP – displacement sensor, F – filters, SH – hydraulic motor,  $S_z$  – actuator,  $T_{emp}$  – temperature transducer, SW – clutch,  $T_{eks}$  – torque transducer,  $n$  – angular velocity sensor, SE – electric motor,  $Z_b$  – oil reservoir.

### 3.2 Object of Research

The subject of the study was a clutch consisting of steel discs. The discs were made of relatively soft S355 steel. The faces of the discs were ground and their roughness was less than  $R_a < 1.25$  at the start of the research. Splines were manufactured according to DIN 5480 standard. The friction discs had 22 teeth and their module were 1.5 mm, while separator discs had 45 teeth and module equal to 3 mm. The internal diameter of separator discs were equal to 80 mm ( $R_i = 40$  mm), while external diameter of friction discs were



**Fig. 6.** Characteristics of torque transmitted by multi-disc wet clutch:  $R_i = 40$  mm,  $R_o = 57.5$  mm. Number of friction surfaces: a) 18; b) 16; c) 14; d) 12; e) 10.

115 mm ( $R_o = 57.5$  mm). Therefore, alternately mounted separator and friction discs formed annular friction area between of internal diameter 80 mm and outer diameter 115 mm. Thickness of all discs were equal to 2 mm.

### 3.3 Results

Results of experiments  $T_{eks}$  and characteristics obtained based on Eqs. (1), (4) and (5) are shown in Fig. 6. The characteristics are illustrated as functions of average pressure  $p$  (Eq. (6)). The Fig. 6 includes also equations describing characteristics  $T_{eks}$ ,  $T_{os\_max}$ ,  $T_t^a$ . Characteristics  $T_{os\_max}$  were calculated with Eq. (4). For analysis purposes it was assumed that value of friction coefficient was maximum i. e.  $\mu = 0.12$  [15]. The same value of friction coefficient was assumed for coefficients  $\mu_o$  and  $\mu_{in}$ . The lowest value of correlation coefficient is 0.89 (Fig. 6a). It indicates very strong correlation between results of experiments and trend line  $T_{eks}$ .

## 4 Discussion

All characteristics shown in Fig. 6 are linear. It can be seen in all figures that the  $T_{eks}$  torque reach significantly higher values than the  $T_{os\_max}$ . Therefore, it is indisputable that design of clutch based on Osinski model (Eq. (4)) might lead to significantly oversized multi-disc clutch. It might happen even if the highest values of friction coefficient  $\mu$  are assumed. Similarly, the  $T_t^a$  functions reach noticeably higher values compared to the  $T_{os\_max}$  characteristics. Figure 6a shows that the  $T_{eks}$  and  $T_t^a$  functions have similar values. It demonstrates a very good correlation between the mathematical model (Eq. (5)) and the experimental results.

It can be seen in other diagrams that differences between slope angle of the  $T_t^a$  and  $T_{eks}$  characteristics are minimal. However, the  $T_{eks}$  functions are shifted upwards with respect to  $T_t^a$ . Despite a noticeable scatter of measurement points for low values of pressure  $p$ , which difference reach several tens of Nm, a very good agreement between the  $T_{eks}$  characteristic and the experimental results is obtained.

It is presumed that positive values of  $T_{eks}$  for  $p = 0$  MPa (Fig. 6b–e) are caused by roughness of adjacent discs and adhesive forces between discs [11].

## 5 Conclusion

The article includes mathematical models which describe torque capacity of a clutch. The paper also involves graphs showing both experimentally and analytically obtained characteristics of torque transmitted by mutli-disc wet clutch. The test stand and methodology of experiments are briefly described in chapter 3.

Every configuration of tested multi-disc clutches indicate that the model proposed by Båk gives better accuracy than previously known models (Eq. (1), Eq. (4)). Therefore this model ought to be taken into account during clutch design process.

Nevertheless, it is necessary to conduct further tests in order to verify correspondence between the model and results of experiments. Deformations of discs should be

taken into consideration, as well as their thickness. In order to reduce torque  $T_{eks}$  for  $p = 0$  MPa which reach significant values it is recommended to assess and replace all clutch discs after each serie of an experiment. As a result of such actions wear of discs should be minimalized. Consequently, differences between results of experiments for approximately the same contact pressure  $p$  should be reduced as well.

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