VILNIUS GEDIMINAS TECHNICAL UNIVERSITY

FACULTY OF FUNDAMENTAL SCIENCES

DEPARTMENT OF MATHEMATICAL STATISTICS

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STRESS TESTING IN CREDIT RISK ANALYSIS

KREDITO RIZIKOS VERTINIMAS TESTUOJANT NEPALANKIOMIS SĄLYGOMIS

Final master project

Applied Statistics program, state code 62401P203

Statistical methods in finance and economics specialization

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MATEMATINĖS STATISTIKOS KATEDRA

TVIRTINU
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Apžvelgti pagrindinius kredito rizikos vertinimo metodus bei kredito rizikos vertinimą esant nepalankioms ekonominėms sąlygoms. Remiantis empiriniais duomenimis, įvertinti kredito riziką esant nepalankioms sąlygoms.

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Annotation

The supervising institutions do not give to commercial banks indications what models have to be used for stress testing. This research was done in order to find out which mathematical/statistical models are and can be used in credit risk stress testing. Credit risk is one of the biggest financial risks that every bank faces.

Stress testing is a tool of credit risk assessment that helps to estimate the consequences of the events that have really small probability to happen but if they occur, banks can have significant losses. This study determined that the most plausible event is adverse macroeconomic conditions. For this reason, models that include macroeconomic impact were presented. Vector autoregression and vector error correction model were tested using the empirical data received from Swedish central bank, Swedish statistics and Eurostat.

For financial stability it is worth using vector autoregression or vector error correction model as they describe the macroeconomic environment in the most suitable way and they are appropriate for shock analysis by showing how the impact of any factor can change the whole system.

Structure: introduction, main part (credit risk, methods and empirical analysis), publication, conclusions, references.

Thesis consists of: 50 p. text without appendices, 13 pictures, 11 tables, 26 bibliographical entries.

Appendices included.

Keywords: credit risk, stress testing, vector autoregression, vector error correction model

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Pavadinimas: **Kredito rizikos vertinimas testuojant nepalankiomis sąlygomis**

Autorius **G. Ramanauskaitė** Vadovas **prof. habil. dr. Henrikas Pragarauskas**

Kalba

	Lietuvių
x	Užsienio (anglų)

Anotacija

Kredito įstaigų priežiūros institucijos nepateikia komerciniams bankams kokius metodus jie turėtų naudoti testavime nepalankiomis sąlygomis. Tiriamasis darbas buvo atliktas tuo tikslu, kad būtų išsiaiškinta kokie matematiniai ir statistiniai metodai yra ir gali būti naudojami kredito rizikos vertinime testuojant nepalankiomis sąlygomis. Kredito rizika yra viena iš didžiausių finansinių rizikų su kuria bankai susiduria.

Testavimas nepalankiomis sąlygomis yra kredito rizikos vertinimo įrankis, padedantis nustatyti įvykių, kurių realizavimosi tikimybės yra mažos, tačiau jiems įvykus, bankai patirtų reikšmingus nuostolius, pasekmes. Šis tyrimas nustatė, jog labiausiai tikėtinas įvykis gali būti ypatingai nepalankios ekonominės sąlygos. Dėl šios priežasties darbe yra pristatyti metodai, kurie įvertina makroekonominių veiksnių įtaką. Vektorinė autoregresija ir vektorinis paklaidų korekcijos modelis buvo patikrinti naudojant Švedijos centrinio banko, Švedijos statistikos departamento ir Eurostat empirinius duomenis.

Finansinio stabilumo įvertinimui vertėtų naudoti vektorinį autoregresijos ar vektorinį paklaidų korekcijos modelius, nes šie modeliai geriausiai aprašo ekonominę aplinką bei yra labai tinkami šokų analizei, kadangi įvertina bet kurio veiksnio įtaką visai sistemai.

Struktūra: įvadas, pagrindinė dalis (kredito rizika, metodai ir empirinė analizė), publikacija, išvados, literatūros sąrašas.

Tiriamasis darbas sudarytas iš: 50 psl. teksto be priedų, 13 paveikslų, 11 lentelių, 26 literatūros šaltinių įrašų.

Priedai pridedami.

Raktiniai žodžiai: testavimas nepalankiomis sąlygomis, VAR modelis, VEC modelis, makroekonominių veiksnių įtaka

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Introduction

Globalization and the introduction of state-of-the-art technology in financial markets have helped to improve efficiency of markets. However, they have also helped to spread shocks rapidly across markets. It means that any country can suddenly get negative impact from another country's problems. In order to be prepared for such events in advance just in case if they happen, banks are using stress testing. Stress testing is modeling of exceptional but plausible events, which are usually accompanied by rather abnormal behaviour of the macroeconomic variables. This procedure allows measuring the possible loss that can be suddenly influenced by adverse macroeconomic environment. Stress testing is assuming an increasingly important role in the macro-prudential analysis of public authorities.

Credit risk loss in the case of negative macroeconomic development is concerned in this study, because financing is still the main business of commercial banks and, as a result, credit risk is still the biggest of all financial risks. Actually, stress testing can be used in any financial risk analysis. The models that create links between the credit risk measures and macroeconomic variables are presented. For the empirical analysis, a time series of Swedish corporate probabilities of default and macroeconomic data of Sweden and Europe were used.

Risk is commonly described as being of two types: specific and non-specific. The latter is also called market or systematic risk. Specific risk is the component of risk associated with a sector of the market, whereas non-specific risk associated with factors affecting the whole market.[1] It is important to note that the models captured only the systematic risk of all companies.

The aim of the study was to explore which statistical and mathematical methods are suitable for credit risk stress testing. In order to reach this aim it was formulated these tasks:

- to analyze literature about credit risk, its assessment and general credit risk models;
- to conclude findings with separating models and important information about stress testing;
- to find out statistical and mathematical methods that can be suitable for credit risk stress testing;

[1] Wilmott P., Howison S., Dewynne J. (1996) The Mathematics of financial derivatives: a student introduction, p. 34;

- to select macroeconomic variables with significant influence to credit risk metric;
- empirical verification of several credit risk stress testing models.

The object of the project was the corporate probability of default, its dependence from the macroeconomic variables and its testing in adverse economic environment.

The first chapter and the first sections of it introduced the reader to credit risk, its assessment and credit risk models. At the end of the starting chapter general information about stress testing is given. As stress testing is still in its infancy, techniques for its implementation are not entirely coherent or uniform. Thus, the project attempts to provide some suggested structure to the stress testing process by considering its two broad dimensions: sensitivity and scenario testing. This study does not capture the second-round effects of a stress scenario that requires the analysis of feedback effects both within the financial sector (such as interbank linkages) and between the financial sector and the real economy. This study provides methods that can be used in credit risk stress testing. All about these methods are in the second chapter.

The second chapter describes the most popular model used in world banks that is macroeconometric CreditPortfolioView model. The model was proposed by Wilson (1997), adjusted by Boss (2002) and other scientists and risk analysts, the newest version of this model was presented in this paper. CreditPortfolioView had been applied for Finish banks' corporate credit portfolio by Virolainen (2004), Canada bank had presented the recommendation work for Canada banking system by proposing this model either. The next model that was explained is based on Merton. The version of the model that includes macroeconomic impact was clarified. In general this model is known as a general credit risk model. The last but not least the vector models (vector autoregression and vector error correction model) were presented, as these models describe the macroeconomic environment in the most suitable way, it takes some macroeconomic variables as endogenous (national factors) and the other as exogenous (regional or global). These kinds of models are appropriate for shock analysis, as it finds out how the impact of any factor can change the whole system. In order to learn how it works with real data, in the next chapter empirical analysis was made.

The third chapter provides the realizations with real data, where vector autoregressive and vector error correction models were tested with Sweden central bank, Swedish statistics and Eurostat empirical data. The models described EDF quite well just in the case when not to many variables were used. As the simplest result it became completely clear that all models are based on econometrical analysis. So even for simple stress testing it would be possible to construct an unsophisticated regression equation. However, the simple regression model should be used just at the beginning to feel how stress testing works. More sophisticated models as CreditPortfolioView, Merton with macro variables, vector autoregression or vector error correction models have to be used after trying an undemanding regression. Moreover, some expert judgment is needed for scenario analysis in order to find the best extreme values for stress testing. Empirical analysis included even the values taken as five standard deviations from the mean, although the result of credit risk parameter was not as shocked as it would have exceeded the historical values.

In the fourth chapter the article was included that is published as a material for the conference No 11th of Lithuanian junior scientists called "Science – the future of Lithuania" that took part March 27th, 2008 in Vilnius.

The last chapter concludes the whole research with the main findings and recommendations for the future research and practical applications of credit risk stress testing.

Most supervision agencies of financial institutions have the objectives that are to strengthen and deepen financial systems and enhance their resilience, i.e. reducing the potential for systematic crises, limiting the severity of crises, addressing structural weaknesses. Risk and vulnerability are identified using both quantitative tools and qualitative assessments. One of the key quantitative tool in financial stability assessment is stress testing.[2] One of the main conclusions is that for financial stability it is worth using vector autoregression and vector error correction model. The research can be expanded for empirical applications, as it has strong theoretical part, but it needs to be improved with the practical implementation. This was not made because of the restrictions of getting credit risk data.

[2] Swinburne M. (2007) The IMF's Experience with Macro Stress-Testing. ECB High Level Conference on Simulating Financial Instability Frankfurt.

1. Credit risk

1.1 Credit risk measurement

Credit risk arises from all transactions that give rise to actual, contingent or potential claims against any counterparty, obligor or borrower. The worst consequence of credit risk to financial institution is default. Default occurs when counterparties fail to meet contractual payments obligations.

There are two metrics required to quantify credit risk. The first metric is called Expected Loss (EL). This is an expectation it is not risk and should be built into the cost of a transaction. The second metric gets to the heart of credit risk and is referred to as Economic Capital (EC) and it is called Unexpected Loss (UL). Where EL measures the anticipated average loss from a portfolio over the relevant time horizon, EC captures the variance or the uncertainty of the losses around the average. With its focus on uncertainty, EC quantifies the portfolio credit risk. These concepts are depicted in the loss distribution presented in Figure 1.

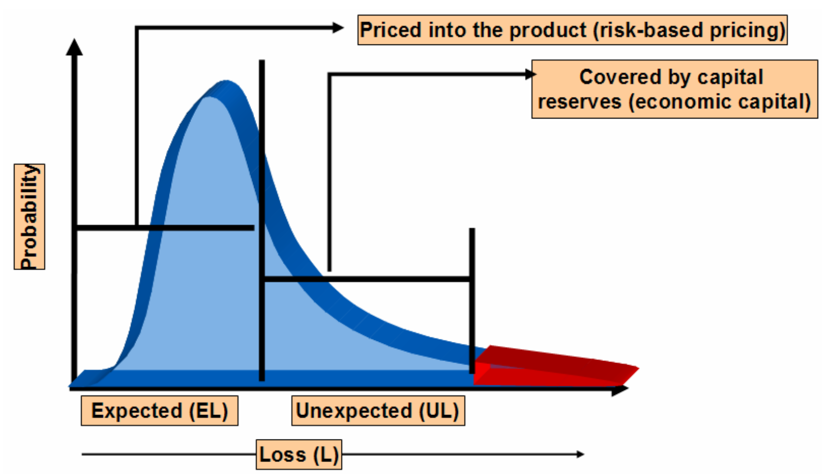

Figure 1 Loss distribution of credit risk (Ravishankar D., 2003)

Expected Loss is measured by multiplying together three factors: PD, LGD and EAD. PD is probability of default, LGD is loss given default and EAD is exposure at default. PD is counted for one year time horizon, estimated with rating-system or scoring-model. LGD is more understood as the converse of the recovery ratio. EAD is loan amount. The measure of EL is given in Figure 2.

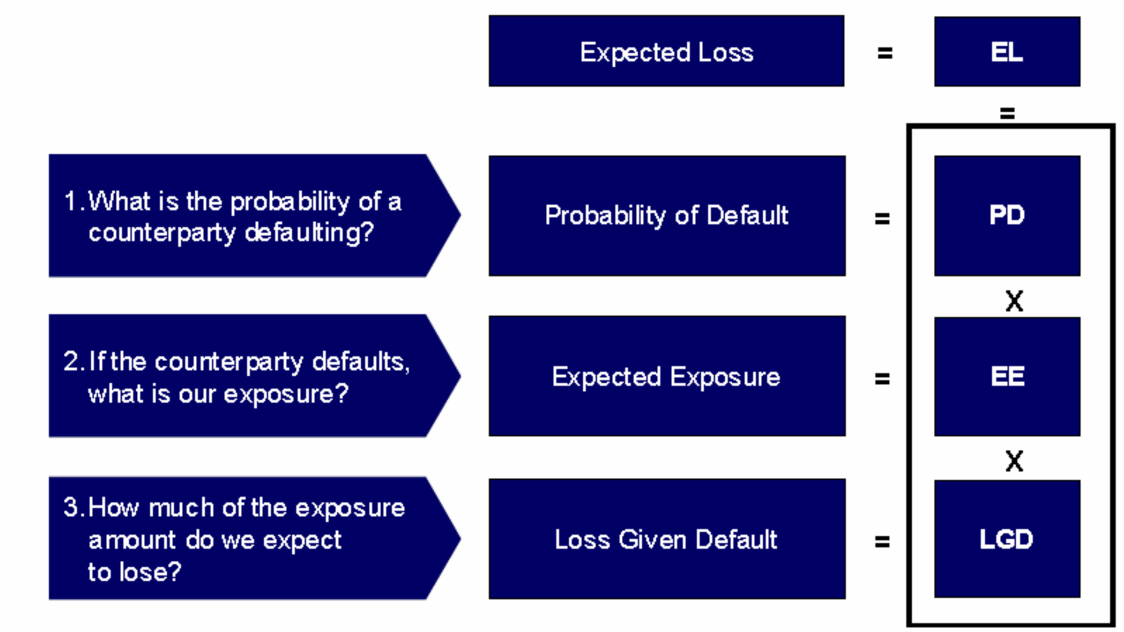

Figure 2 Measuring expected loss (Rich J., Tange C., 2003)

Expected loss is counted by multiplying all the factors described above

$$EL = PD \times LGD \times EAD\,. \tag{1}$$

Unexpected Loss is the estimated volatility of the potential loss in value of the asset around its expected loss[3]

$$UL = Var(\xi)\,, \tag{2}$$

where ξ is a stochastic variable, whose $E\xi = EL$.

The distribution of possible future losses for a portfolio of credit risky corporate assets shows strongly asymmetric behavior and a fat tail as the consequence of the limited upside of credit and substantial downside if the corporation defaults (Figure 1). Because of correlation, it is not possible to fully diversify away this fat tail. It is obvious from portfolio theory that the portfolio unexpected loss is not equal to linear sum of the individual unexpected losses of the risky assets that make up the aggregate portfolio

$$UL_P << \sum_i UL_i\,. \tag{3}$$

[3] Ong M.K. Capital allocation and performance measurement. *Internal Credit Risk Models*. p. 112;

This implies that only a portion of each asset's unexpected loss actually contributes to the portfolio's total risk of loss. This portion is called the risk contribution (RC). From a portfolio management perspective, RC is the single most important risk measure for assessing credit risk. The risk contribution of a risky asset (RC_i) to the portfolio unexpected loss is defined as the incremental risk that the exposure of a sinlge asset contributes to the portfolio's total risk. Mathematically, *RC* of asset *i* is written

$$RC_i \cong UL_i \frac{\partial UL_P}{\partial UL_i} . \tag{4}$$

From equation (4) it is seen that RC_i is a sensitivity measure of UL_P with respect to the UL_i. In practice this equation is used so follows

$$RC_i = \frac{UL_i \sum_j UL_j \rho_{ij}}{UL_P} , \tag{5}$$

where ρ_{ij} is the pairwise correlation of default between asset *i* and asset *j*. The correlation of default is the glue that binds all of the risk contributions of the individual assets to the portfolio as a whole.

The risk contribution is a measure of the undiversified risk of an asset in the portfolio. The sum of all the risk contributions from all the assets in the portfolio is the portfolio unexpected loss

$$UL_P = \sum_{i=1}^{N} RC_i . \tag{6}$$

From a credit risk management and measurement perspective, the following three issues are equivalent:

- correlation of default;
- concentration risk;
- diversification.

The level of concentration risk decides on the degree of diversification in the portfolio.[4]

1.2 Credit risk models

Credit risk modeling has gained increasing impetus among bankers and other portfolio managers since the mid 1990s. Among the better known publicly available models, there are four types:

- Merton-based, e.g. KMV's Portfolio Manager;
- Ratings-based, e.g. The RiskMetrics Group's CreditMetrics;
- Macroeconomic, e.g. McKinsey's CreditPortfolioView;
- Actuarial, eg CSFP's CreditRisk+.

Merton-based

There are based on the model of a firm's capital structure first proposed by Merton 1974: a firm is considered to be in default when its assets falls below that of its liabilities. The magnitude of the difference between the assets and liabilities and the volatility of the assets then determine the borrower's default probability. KMV has developed an extensive database to assess the loss distribution related to both default and credit quality migration. KMV's Credit Monitor calculates an expected default frequency (EDF) for each individual borrower as a function of the firm's capital structure, the volatility of its asset returns and its current asset value, using Merton's contingent claim model. KMV's historical data are then used to derive loss estimates.

Ratings-based

CreditMetrics assumes that changes in a latent variable which drives credit quality are normally distributed. The probability of a borrower's change in credit quality (including default) within a given time horizon can be expressed as the probability if a standard normal variable falling between various critical values. These critical values are calculated using the borrower's current credit rating and historical data on credit rating migrations. They are generally presented in the form of a matrix of probabilities that a borrower with one rating might move into another rating category during a year. For example, for an A-rated credit one row of matrix shows the probabilities that its rating will change to AAA, AA, BBB, BB, or C, or that the obligor will default; the closer the rating category to current rating, the higher the probability of a move to that category. Both Merton-based and ratings-based models convert the estimates of losses on individual credits to estimates of loss on whole portfolio by estimating the correlations in changes in credit quality for all pairs of obligors. Both CreditMetrics and KMV's

[4] Ong M.K. Capital allocation and performance measurement. *Internal Credit Risk Models*. p. 125-127, 135;

PortfolioManager make the simplifying assumption that a firm's asset returns are generated by a set of common, or systematic risk factors along with idiosyncratic factors. The idiosyncratic factors may be firm specific, country specific or industry specific.

Macroeconomic

The most widely used of these, CreditPortfolioView, measures only default risk, and attempts to take into account the link between default probabilities in any period and the macroeconomic environment. It uses Monte Carlo simulation to estimate the joint distribution of default probabilities for individual credits conditional on the value of macroeconomic factors such as the unemployment rate, the growth rate of GDP, the level of long-term interest rates, foreign exchange rates, government expenditure and the aggregate savings rate. Correlations between default rates for different obligors are considered to arise from the covariance structure of the underlying macroeconomic variables.

Actuarial

CreditRisk+ estimates the loss distribution using statistical techniques developed in the insurance industry. Only default risk is considered. Rather than attempting to relate this to the structure of the firm, the model allocates the borrowers amongst „sectors", each of which has a mean default rate and a default rate volatility. Default for individual loans is assumed to follow a Poisson process. Although credit migration risk is not explicitly modeled, CreditRisk+ assumes that the mean default rate is itself stochastic. This assumption generates a skewed distribution of default events, which is taken to account (if only partially) for migration risk.

As later in this paper KMV model's credit risk measure is used, so the main idea and background of this model is going to be expatiated thereinafter. Moreover, CreditPortfolioView will be used as well, but this model is described in the Section 2.

Moody's KMV

Moody's KMV calculates Expected Default Frequency (EDF) – an objective, forward-looking probability of default measure – by compiling about a firm's equity, leverage, industry, volatility, financial statement data, and historical defaults, and by performing an analysis using the advanced financial model.[5] In simple words EDF is the probability that a firm will default within a given time horizon. For example, a company with a current EDF credit measure of 5% has a 5% probability of defaulting within the next twelve months. EDF is just for firms with

[5] http://www.creditedge.com/ (watched 2008-04-10)

publicly traded equity. Moody's KMV[6] is based on Merton model. Graphical representation of Merton model is shown on Figure 3.

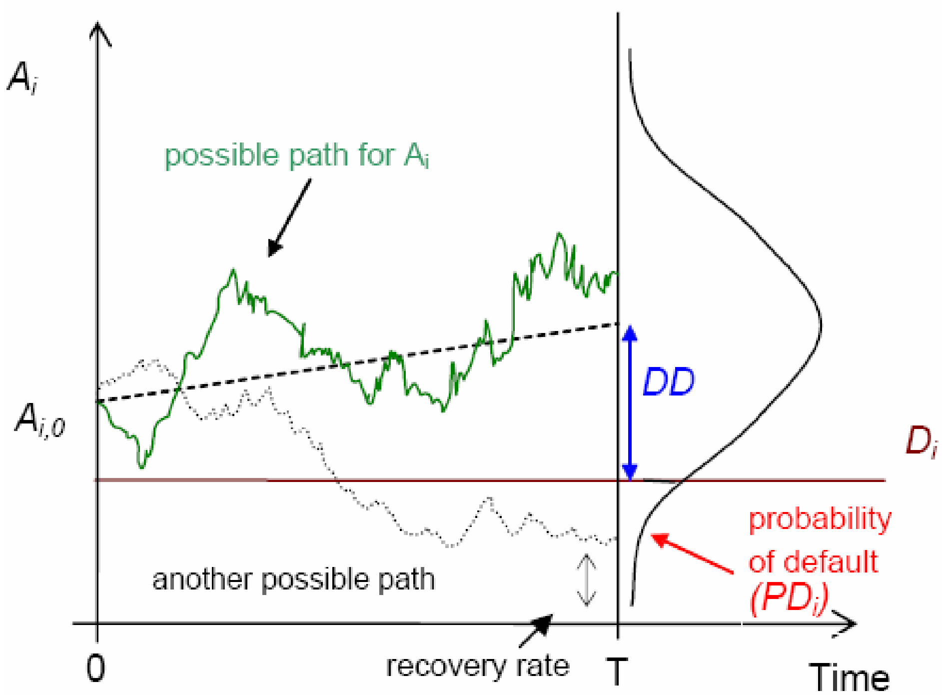

Figure 3 Merton model (Drehmann M., 2005)

A_i is the market value of i firm's assets and its dynamics is stochastic process with the drift μ_i and volatility σ_i, $i = 1,\ldots n$

$$dA_i = \mu_i A_i dt + \sigma_i A_i dz \,, \tag{7}$$

where z is a Brownian motion. In other way the same can be written

$$A_{it} = A_{i0} e^{(\mu_i - \frac{1}{2}\sigma_i^2)t + \sigma_i z_t} \,, \tag{8}$$

where A_{it} is the market value of i firm's assets at the time t.

When the value A_i gets lower than the liabilities' level D_i (usually fixed at the level of short-term liabilities plus half of long-term liabilities), then it means that the firm defaults. Mathematically probability of default is

[6] This model is based on Merton's approach for the evaluation of credit risk as refined by Vasicek and Kealhofer, which is why it is known as Kealhofer Merton Vasicek (KMV);

$$PD_i = \mathrm{P}\{A_{it} < D_i\}, \tag{9}$$

here t < T, T is maturity time. Quite often probability of default (PD_i) is identified as the distance to default noted DD_i.[7] Moody's KMV Credit Monitor calculates EDF as a function of DD_i: $EDF=f(DD_i)$. DD_i is defined as follows

$$DD_i = \frac{A_i - D_i}{\sigma_i}, \tag{10}$$

here A_i is the market value of company's i assets, D_i is company's i debts and σ_i is the volatility of the market value of the company's i assets.[8] Market value of the company's assets can be calculated using option pricing theory.[9]

Option pricing

European call option is the right to buy share with the strike price K at the moment T. The value of this call option at the time t = 0 is

$$C_T = S_0 \Phi\left(\frac{\log \frac{S_0}{K} + T(r + \frac{\sigma^2}{2})}{\sigma\sqrt{T}} \right) - Ke^{-rT} \Phi\left(\frac{\log \frac{S_0}{K} + T(r - \frac{\sigma^2}{2})}{\sigma\sqrt{T}} \right),$$

where Φ is the standard normal distribution function.

At any time t (t < T) the value of the call option is

$$C_{T-t} = S_t \Phi\left(\frac{\log \frac{S_t}{K} + (T-t)(r + \frac{\sigma^2}{2})}{\sigma\sqrt{T-t}} \right) - Ke^{-r(T-t)} \Phi\left(\frac{\log \frac{S_t}{K} + (T-t)(r - \frac{\sigma^2}{2})}{\sigma\sqrt{T} - t} \right).$$

[7] Drehmann M. 2005. A Market Based Macro Stress Test for the Corporate Credit Exposures of UK Banks, p. 9;
[8] Asberg P., Shahnazarian H. (2008) Macroeconomic Impact on Expected Default Frequency. *Sveriges Riksbank working paper series 219*, p. 6, 27;
[9] Leipus R., Valužis M. (2006) Kredito rizika kaip pasirinkimo sandoris. *Pinigų studijos 2006/1*, p. 38;

Equity derives its value from the cash flows of the firm. Equity is a call option on the firm's assets: the right, but not the obligation, to "buy" the firm's assets from the lender by re-paying the debt.[10]

Table 1 Calculating Market Value of Assets (Moody's KMV, 2004)

Standard Options Terms		KMV approach
Call Option Value	=	Market Value of Equity
Strike Price	=	Book Liabilities
	↓	
Implied Underlying Asset Value		Implies Market Value of Assets

In KMV approach the market value of equity is replaced by C_t, book liabilities are replaced by K, and the market value of the assets is implied instead of S_t.

Graphical explanation of Moody's KMV is presented in Figure 4.

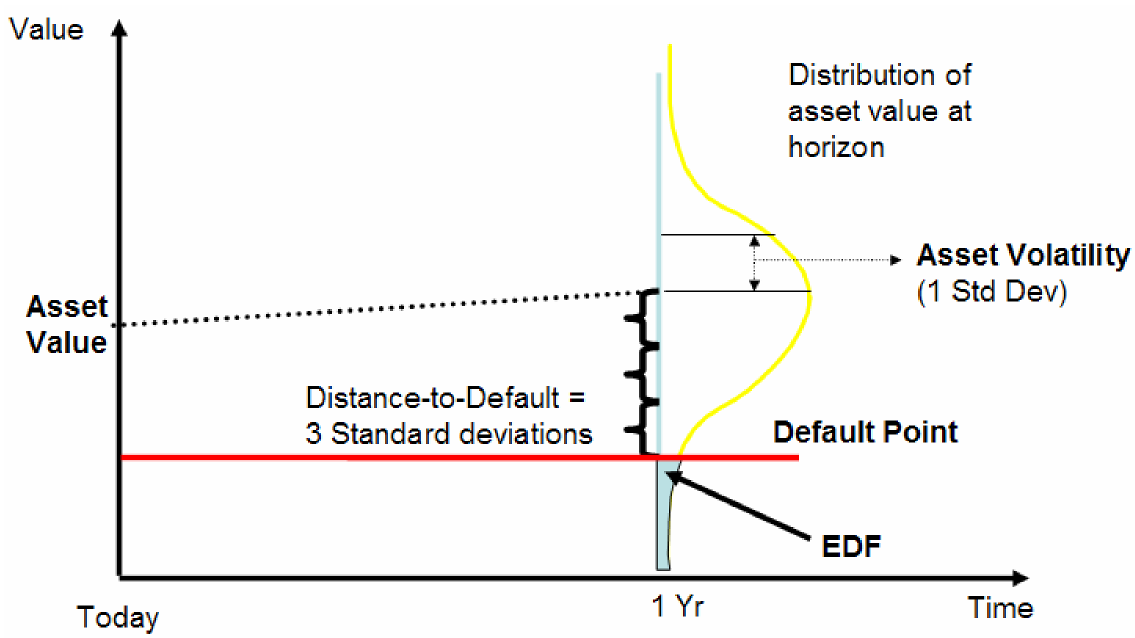

Figure 4 Moody's KMV (Moody's KMV, 2004)

In Figure 4 it is just an example that DD_i is three standard deviations.

Moody's KMV EDF model is used by hundreds of banks, insurance companies, asset managers and the other to measure credit risk for both financial and industrial firms. In the model good times are anticipated by rising asset values, and downturns by declines in asset values.[11]

[10] (2004) Measuring & Managing Credit Risk: Understanding the EDF Credit Measure for Public Firms. *Moody's KMV*, p. 15;

1.3 Stress testing

While value-at-risk, calculated on daily basis, supplies forecasts for maximum losses under normal market conditions, simulating extreme market movements using stress tests, in which valuing portfolio loss under extreme market scenarios not covered by value-at-risk. Large-than-average losses due to default and bankruptcies are usually attributed to adverse economic conditions. Stress-tests at the level of individual institutions have been widely applied by internationally active banks since the early 1990s. They are generally used to complement financial institutions' internal models, as recommended also by the Basel II capital adequacy framework.[12] Stress testing is the process of determining the effect of a change to a portfolio or sub-portfolio due to extreme, realistic events.[13] Stress testing results is a "Worst-Case" loss part of unexpected loss (Figure 5).

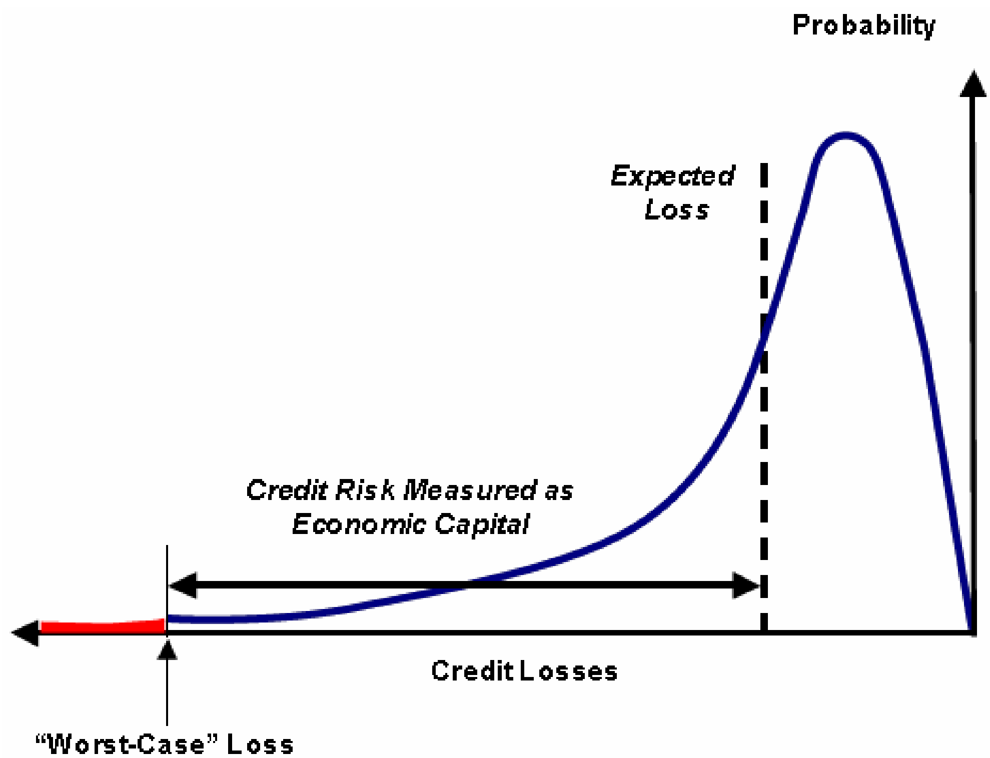

Figure 5 Loss distribution and the metrics of credit risk (Rich J., Tange C., 2003)

Stress testing is a generic term used to describe various techniques and procedures employed by financial institutions to estimate their potential vulnerability to exceptional but plausible event. An event couldn't be described as a stress event unless it's "Exceptional" – not

[11] Sellers M., Arora N. (2004) Financial EDF Measures. A new model of dual business lines, p. 5;
[12] Sorge M., Virolainen K. (2005) A comparative analysis of macro stress-testing methodologies with application to Finland. p. 2;
[13] Henbest J. (2006) Stress Testing: Credit Risk. *Algo Capital Advisory Algorithmics, Inc.* p. 5;

extreme – (with low probability to happen) but yet “Plausible” event to guarantee the significance of the test results.

A sophisticated simulation-based stress test is a conditional simulation, i.e., constructing a conditional loss distribution based on constraining simulation to those consistent with defined stress scenarios. The result would be portfolio and obligor losses conditional on those constraints, e.g., downturns in that set of countries and/or industries (Figure 6).

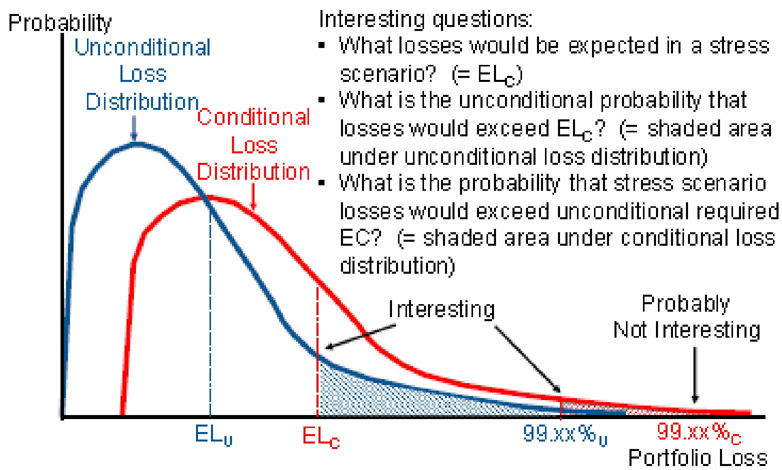

Figure 6 Conditional simulation (Dvorak B., 2008)

Figure 6 illustrates, the conditional loss distribution in a stress scenario would be expected to be shifted to the right of the unconditional loss distribution. Stress testing may help a financial institution determine whether it is adequately capitalized. In particular, portfolio losses would need to be calculated in some direct manner, analogous to calculating portfolio losses for a single Monte Carlo scenario in a credit portfolio model, which would require specifying both systematic as well as obligor-specific shocks in the scenario. Also, it may be very difficult to estimate the probabilities of the scenarios accurately.[14]

Main types of credit stress tests:

- Sensitivity analyses (single factor shock);
- Scenario analyses (multi-factor shock).

Sensitivity analysis is worth to use when the impact of a large movement on single factor of the model wanted to be evaluated. Scenario analysis gives the full representations of possible future situations to which portfolio may be subjected.

[14] Dvorak B., (2008) Credit Portfolio Stress Testing and Scenario Analysis. p. 5-7;

Designing useful and meaningful stress tests and scenarios can be challenging. Concerning the stress scenarios, they are generated by *historical events*, by doing forward assessment of developing market trends or by making *hypothetical scenarios*.[15] Historical events are based on observed event from the past that is actual events, less subjective but may be irrelevant. Hypothetical events are plausible events that are yet to be realized, they are more relevant, but requires expert judgment and analysis.

Constructing a good scenario involves determining the overall impact by adjusting a set of variables that can influence the output and estimating the probability of occurrence. A useful scenario is realistic, corresponds to the approach and portfolio of exposures, is informative and valuable to risk management objectives.[16]

There are at least three key challenges in designing stress scenarios:

1. to create scenarios that are both plausible and stressful enough for their intended purpose;
2. to calculate the obligor and portfolio losses that would result from the stress scenario;
3. to estimate the probability of the scenario occurring.[17]

The features of good stress-testing programs and the tools required conducting them, Table 2 suggests the procedure for constructing a stress-testing program.

Table 2 A stress-testing program (Enoch C., 2003)

<table>
<tr><td colspan="2">Ensure reliable data</td></tr>
<tr><td colspan="2">Survey Portfolio & Environment</td></tr>
<tr><td colspan="2">Identify risk factors</td></tr>
<tr><td colspan="2">Construct Stress-tests</td></tr>
<tr><td colspan="2">Decide magnitude of factor shock</td></tr>
<tr><td colspan="2">Does the bank possess quantitative risk measurement systems?</td></tr>
<tr><td>YES</td><td>NO</td></tr>
<tr><td>Run Stress-tests using obligor and portfolio risk models</td><td>Re-estimate bottom-line of obligors under stressful conditions</td></tr>
<tr><td colspan="2">Report results</td></tr>
<tr><td colspan="2">Take Corrective Action, if required</td></tr>
<tr><td colspan="2">Reassess relevance of Stress-tests to new environment/portfolio</td></tr>
</table>

[15] Stress Testing Best Practices & Risk Management Implications for Egyptian Banks. *Seminar material*, p. 1;
[16] Henbest J. (2006) Stress Testing: Credit Risk, p. 7, 9;
[17] Dvorak B., (2008) Credit Portfolio Stress Testing and Scenario Analysis. p. 9-10;

For useful stress testing it is important to segment the portfolio (e.g. by sectors), identify risk factors (can be unique for each sector) to be stressed, and, finally, analyze stress test results (compare with original distribution, sensitivity). The most common type of stress test examines industries or sectors. Another direction for doing stress testing, involving somewhat more complex tools, would be to have them based on macroeconomic factors, such as interest rates, energy, or unemployment.

1.4 The core of the problem

Stress testing is a tool for all financial risks. However, credit risk takes the biggest part in the banks. For this reason it was decided to analyze credit risk stress testing. Moreover, supervising institutions do not provide commercial banks with the methods that have to be used for this part of credit risk assessment. Every bank uses each own methods, some of them just make reports in order to deliver them to the supervising institutions without concerning the usefulness of the model. Thereby, in this research it was concerned what methods can be used for stress testing in credit risk analysis.

Most stress scenarios tend to be macroeconomic in nature. The macroeconomic factors describing the stress scenario therefore need to be related to these systematic risk factors in order to estimate the resulting obligor and portfolio losses. This requires economic and econometric expertise. The probability can be estimated if the scenario can be described in terms of macroeconomic factors and if the joint distribution of future realizations of those factors can be estimated.[18]

Thus, the methods of stress testing were concerned in the way that it would be possible to evaluate the impact of macroeconomic environment to credit risk.

The next chapter presents the models that can be used in credit risk stress testing. One of them is the most popularly used in foreign banks that is constructed for each sector separately, which also helps to find the relation between the sectors and gives possibility to measure the impact of macroeconomic variables. The next one is based on Merton and the last one type of models has been recently researched as useful models for financial stability. Some of the models will be tested with empirical data.

[18] Dvorak B., (2008) Credit Portfolio Stress Testing and Scenario Analysis. p. 10;

2. Methods

2.1 CreditPortfolioView model

CreditPortfolioView model for macroeconomic stress testing was developed by Wilson (1997), Boss (2002) and Virolainen (2004). [19] The idea is to model the relationship between default rates and macroeconomic factors, and when model is fitted, to simulate the evolution of default rates over time by generating macroeconomic shocks to the system.[20]

The framework comprises:

- an empirical model with a system of equations on credit risk and macroeconomic dynamics, and
- a Monte Carlo simulation for generating distribution of possible default rates (or credit losses).[19]

2.1.1. The system of empirical equations

First, the average default rate for industry *j* is modeled by the logistic functional form as

$$p_{j,t} = \frac{1}{1+e^{y_{j,t}}}, \tag{11}$$

where $p_{j,t}$ is the default rate in industry j at time t, and $y_{j,t}$ is the industry-specific macroeconomic index, which parameters have to be estimated. The logistic functional form is convenient in that $y_{j,t}$ is given by the logit transformation

$$y_{j,t} = \ln\left(\frac{1-p_{j,t}}{p_{j,t}}\right). \tag{12}$$

The logit transformed default rate $\boldsymbol{y}_t = (y_{1,t},..., y_{J,t})$‘ is linearly depending on its lags and on the current and lagged values of macroeconomic factors, i.e.

$$y_t = m + \beta_1 x_t + ... + \beta_{1+s} x_{t-s} + \phi_1 y_{t-1} + ... + \phi_k y_{t-k} + v_t, \tag{13}$$

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[19] Wong J., Choi K., Fong T. (2006) A framework for macro stress testing the credit risk of banks in Hong Kong. *Hong Kong monetary authority quartely bulletin.* p. 27;
[20] Virolainen K. (2004) Macro stress testing with a macroeconomic credit risk model for Finland. *Bank of Finland discussion papers.* p. 11;

where $\boldsymbol{x_t}$ is an $M \times 1$ vector of macroeconomic variables; $\boldsymbol{m}$ is a $J \times 1$ vector of intercepts; $\beta_1,...,\beta_{1+s}$ are $J \times M$ and $\phi_1,...,\phi_k$ are $J \times J$ coefficient matrices; and v_t is a $J \times 1$ vector of disturbances. The characterization of equation (13) explicitly links the default behaviours in the J economic sectors to the macroeconomic conditions. Another part of the equation system in the framework is on the dynamics of the M macroeconomic variables.

$$x_t = n + B_1 x_{t-1} + B_p x_{t-p} + \Theta_1 y_{t-1} + ... + \Theta_q y_{t-q} + \varepsilon_t , \tag{14}$$

where $\boldsymbol{n}$ is an $M \times 1$ vector of intercepts; $B_1,\ldots, B_p$ are $M \times M$ and $\Theta_1,...,\Theta_q$ are $M \times J$ coefficient matrices; and ε_t is an $M \times 1$ vector of disturbances. Equations (13) and (14) together define a system of equations governing the joint evolution of the economic performance, the associated default rates, and their error terms.

In this system it is assumed that $\boldsymbol{v_t}$ and ε_t are serially uncorrelated and normally distributed with variance-covariance matrices $\sum_v$ and $\sum_\varepsilon$ respectively; $\boldsymbol{v_t}$ and ε_t are correlated, with variance-covariance matrix $\sum_{v,\varepsilon}$. Therefore, the structure of the disturbances is as follows:

$$e_t = \begin{pmatrix} v_t \\ \varepsilon_t \end{pmatrix} \sim N(0,\Sigma), \;\; \Sigma = \begin{pmatrix} \sum_v & \sum_{v,\varepsilon} \\ \sum_{v,\varepsilon} & \sum_\varepsilon \end{pmatrix}. \tag{15}$$

Allowing the off-diagonal elements of $\sum_v$, $\sum_\varepsilon$ and $\sum_{v,\varepsilon}$ to be non-zero is desirable. First, influences stemming from factors affecting the dependent variables but not explicitly incorporated in equations (13) and (14) will not be omitted altogether. Secondly, the contemporaneous correlation between the two disturbances in equations (13) and (14) can be captured and the feedback effects of bank performances on the economy can be more accurately assessed.

2.1.2. Monte Carlo simulations and stress tests

Estimated frequency distributions of the horizon-end default rates for each sector corresponding to stress and baseline scenarios are obtained separately from simulating a large number of future joint sector-specific default rates by applying a Monte Carlo method. This is partly governed by the simulated future paths of the macroeconomic variables. The reasonableness of the simulated mixes of macroeconomic variables is supported be the estimated relationships based on historical data. The future default rates are simulated from different future evolutions of the macroeconomic environment and the innovations $\boldsymbol{v_t}$ in equation (13).

The baseline simulations produce an estimated unconditional probability distribution of possible credit losses, without the information about the occurrence of a particular shock. In stressed simulations, as the different future evolutions of the macroeconomic environment and the innovations $\boldsymbol{v_t}$ that the simulated paths involved share the same artificial economic shocks, the estimated distribution is conditional on the occurrence of such shocks. Comparing the conditional loss distribution of the stressed scenario with the unconditional distribution of the baseline scenario provides information on the possible impact of adverse macroeconomic conditions triggered by the shock.

2.2 Merton model and macroeconomic factors

As it is known from the previous chapter, default occurs when value A_i of firm's i assets falls below the value of its liabilities D_i, asset value modeled as a Brownian motion process (log returns normally distributed) and value of liabilities fixed over time.
So value of liabilities calibrated such that

$$PD_i = \mathrm{P}(R_{i,T} < D_i) = \mathrm{N}(D_i).^{21} \tag{16}$$

Discrete normalized logarithmic return process satisfies following equation for every company in the economy

$$R_{it} = \sqrt{\rho}F_t + \sqrt{1-\rho}U_{it}\,, \tag{17}$$

where R denotes normalized logarithmic return of assets for each firm i at time t, F represents normalized logarithmic return in the economy independent on firm at time t, U denotes firm specific return, ρ expresses the correlation between the normalized assets returns of any two borrowers. F and U is assumed standard normal random distributed.[22] F can be explained as the

[21] Stapper G. (2007) Merton Style Factor Model, aspects of implementation & application. *Define consulting*, p. 11-12;
[22] Jakubik P. (2006) Does Credit Risk Vary with Economic Cycles? The Case of Finland. *IES Working Paper: 11*, p. 21;

macroeconomic specific part of return and stands for systematic part, U stands for idiosyncratic part of return.

The probability of default event can be rewritten as

$$P(Y_{it} = 1) = P(R_{it} < T), \tag{18}$$

where Y denotes random variable with the two potential state

$$Y_{it} = \begin{cases} 1 \\ 0 \end{cases}, \tag{19}$$

if $Y_{it} = 1$, then borrower i defaults at time t, T can be assumed as constant or variable depends on time. In second case if this threshold is considered with changing in macroeconomics environment at time

$$T = \beta_0 + \sum_{j=1}^{N} \beta_j x_{jt} \ , \tag{20}$$

where x_j represents j-th macroeconomic indicator and β are constant coefficients. It is a simple linear relation for value of threshold that is considered. The default probability of firm i at time t in case of the constant default threshold at time

$$p_i = P(Y_{it} = 1) = P(R_{it} < T) = P(\sqrt{\rho} F_t + \sqrt{1-\rho} U_{it} < \beta_0) = \Phi(\beta_0), \tag{21}$$

where Φ is the standard normal distribution function. After applying conditional default probability on realization of random factor and macroeconomic indicators, the result is obtained under the assumption that macroeconomic indicators are considered as a part of the factor of assets return independent on firm i at time t. Formally,

$$R_{it} = \alpha F_t + \beta_0 + \sum_{j=1}^{N} \beta_j x_{jt} + \omega U_{it} \ .^{23} \tag{22}$$

[23] Jakubik P. (2006) Does Credit Risk Vary with Economic Cycles? The Case of Finland. *IES Working Paper: 11*, p. 22;

2.3 Vector models

This section represents the linear dynamic vector autoregressive models and their vector error correction forms that can be used for investigation of mutual relationship between default rate and macroeconomics indicators.

2.3.1. Vector Autoregressive Model

Vector autoregressive models (VAR) are often used in the case of dynamic model. These models are able to modeled mutual relationship of times series even in case of time series non-stationary.

$$y_t = B(L) y_{t-1} + \varepsilon_t \,, \tag{23}$$

where $B(L) = B_1 + B_2 L + B_3 L^2 + ... + B_q L^{q-1}$, y_t – vector of endogenous variables, y_t depends on own lags, autoregression.

The vector autoregressive model is commonly used for analyzing the dynamic impact of random disturbances on the system variables. VAR model is particularly suitable for shock analysis, as one variable shock shows the influence to the whole system of variables. The tool to find out that is impulse response functions:

$$E(y_{t+k} - E_{t-1}(y_{t+k})) = B^k \varepsilon_t \,, \tag{24}$$

for more lagged variables

$$E(y_{t+k} - E_{t-1}(y_{t+k})) = B(L)^k \varepsilon_t \,. \tag{25}$$

Problem is that ε_{ym1} and ε_{yn1} may be correlated. The problem can be identified by structural model.

The impulse response functions can be used to produce the time path of the dependent variables in the VAR, to shocks from all the explanatory variables. If the system of equations is stable any shock should decline to zero, an unstable system would produce an explosive time path.

Additionally, vector autoregressive moving-average models (VARMA) are very rarely used in applied macroeconomic work. One likely reason is that the estimation of these models is considered difficult by many researches.[24]

Usually it is worth to include exogenous variables into VAR model, then this model is called VARX.

$$Y_{i,t} = B_{1,i} Y_{t-1} + \ldots + B_{p,i} Y_{t-p} + C_i X_t + \varepsilon_t , \tag{26}$$

where $Y_{i,t}$ are endogenous variables, and X_t – exogenous variables, i is variables number, B and C matrixes of coefficients that have to be estimated.

Matrix representation is

$$y_t = B(L) y_{t-1} + C(L) x_t + \varepsilon_t \; . \tag{27}$$

However, no feedback from variables y_t to x_t, x_{t+1}, …, x_{t+k}.

Issues in VAR modeling

- selection of VAR variables;
- selection of VAR levels or differences;
- selection of VAR lag lengths;
- identification scheme (variables ordering and structural VARs).

VAR should chosen in levels, if all variables are stationary (I(0)), and in first differences, if some variables have a unit root (I(1)) and the series are not cointegrated.

Cointegration is an econometric property of time series variables. If two or more series are themselves non-stationary, but a linear combination of them is stationary, then the series are said to be cointegrated.[25]

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[24] Kascha C. (2007) A Comparison of Estimation Methods for Vector Autoregressive Moving-Average Models, p. 2;
[25] http://en.wikipedia.org/wiki/Cointegration (watched 2008-05-07)

Vector autoregressive model that is used frequently in macroeconomic research can be applied for nonstationary time series if cointegration exists.[26] But then the adjusted form of VAR is applied that is called vector error correction model.

2.3.2. Vector Error Correction Model

A Vector Error Correction Model (VEC) can lead to a better understanding of the nature of any nonstationarity among the different component series and can also improve longer term forecasting over an unconstrained model. VEC is a VAR model with error-correction mechanism based on cointegration relationships between variables. VEC is able to distinguish long-run and short-run dependence.

The VEC(p) form is written as

$$\Delta y_t = \delta + \Pi y_{t-1} + \sum_{i=1}^{p-1} \Phi_i^* \Delta y_{t-i} + \varepsilon_t \,, \tag{28}$$

where Δ is the differencing operator, such that $\Delta y_t = y_t - y_{t-1}$.
Matrix representation

$$\Delta y_t = B(L)\Delta y_{t-1} + \Pi y_{t-1} + \varepsilon_t \,, \tag{29}$$

where Π is coefficients matrix that describes long-run equilibrium (=cointegration) relationships between variables y_t.

VEC as VAR can have exogenous variables either. When deciding on a specific vector error correction model with exogenous variables (VECX), one always has to make choices like, e.g., the number of lags to include, the number of cointegrating relations to assume, the long-run restrictions to impose, and the data-generating processes to adopt for the exogenous variables.

$$\Delta y_t = B(L)\Delta y_{t-1} + \Pi y_{t-1} + Dx_t \,, \tag{30}$$

[26] Jakubik P. (2006) Does Credit Risk Vary with Economic Cycles? The Case of Finland. *IES Working Paper: 11*, p. 17;

where y_t is a vector of endogenous variables, x_t is a vector of stationary exogenous variables and D is the matrix of parameters associated with the exogenous variables.[27]

The advantage of any vector approach is that it offers two channels of impact of a macroeconomic shock on default probabilities: the direct impact of a change in macroeconomic variable on default probability, and an indirect impact via the impact on other macroeconomic variables.[28]

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27 Adam C., Hendry S. (1998) The M1 Vector-Error-Correction Model: Some Extensions and Applications, p. 154;
28 Misina M., Tessier D., Dey S. (2006) Stress testing the Corporate Loans Portfolio of the Canadian Banking Sector. *Bank of Canada*, p. 7;

3. Empirical analysis

Expected default frequency (EDF) of Sweden corporate sector is analyzed in this part and its dependence from Swedish macro variables. Moreover, the Europe macroeconomic impact to Swedish macroeconomic development and EDF are tested either.

3.1 Data

The estimations are based on monthly data. Data on the empirical EDF are from the Sweden Riksbank (central bank), Swedish macro variables are taken from the Swedish statistics webpage and European macro variables are taken from the Eurostat. All data are covering the period from the beginning of 1998 till the end of 2007.

As for empirical analysis is going to be used vector autoregressive model and its adjusted version, i.e. vector error correction model, it was decided to separate macro variables into endogenous (Swedish) and exogenous (European).

Swedish variables:

- **EDF** is expected default frequency of non-financial listed companies of Sweden;
- **DEBT_SE** is Swedish companies' borrowing from credit institutions, annual percentage change;
- **IPI_SE** is Swedish industrial production index (NACE C+D), percent change over 12 months;
- **CPI_SE** is Swedish consumer price index, monthly changes per cent by period;
- **LRY_SE** is long-run yield in percent;
- **OMXS30** is logarithm of the Stockholm exchange market stock price index.

European variables:

- **IPI_EU** is total European industry (excluding construction), growth rate in comparison with the same period of the last year;
- **IR_ECB** is the interest rate set by European central bank;
- **HICP_EU** is harmonized index of consumer price, annual rate of change;
- **USD_SEK** is logarithm of the exchange rate among USA dollar and Swedish crone.

Some of the series were generated by natural log. Most of the variables were not transformed, as it is better to use raw data. Natural logs of OMXS30 and USD_SEK are only used in calculations. Logarithm removed some random fluctuations.

Descriptive statistics of all these variables and correlation matrices of Swedish and European variables are presented in the Appendix A. Descriptive statistics shows the main characteristics of empirical data (Appendix A, A.1), and correlation matrices (Appendix A, A.2) identify how variables are related.

Each variable are presented in a graph, it creates an opportunity to observe which variables can be connected. Figure 7 shows the dynamics of the Swedish indicators, and Figure 8 shows the dynamics of the European indicators.

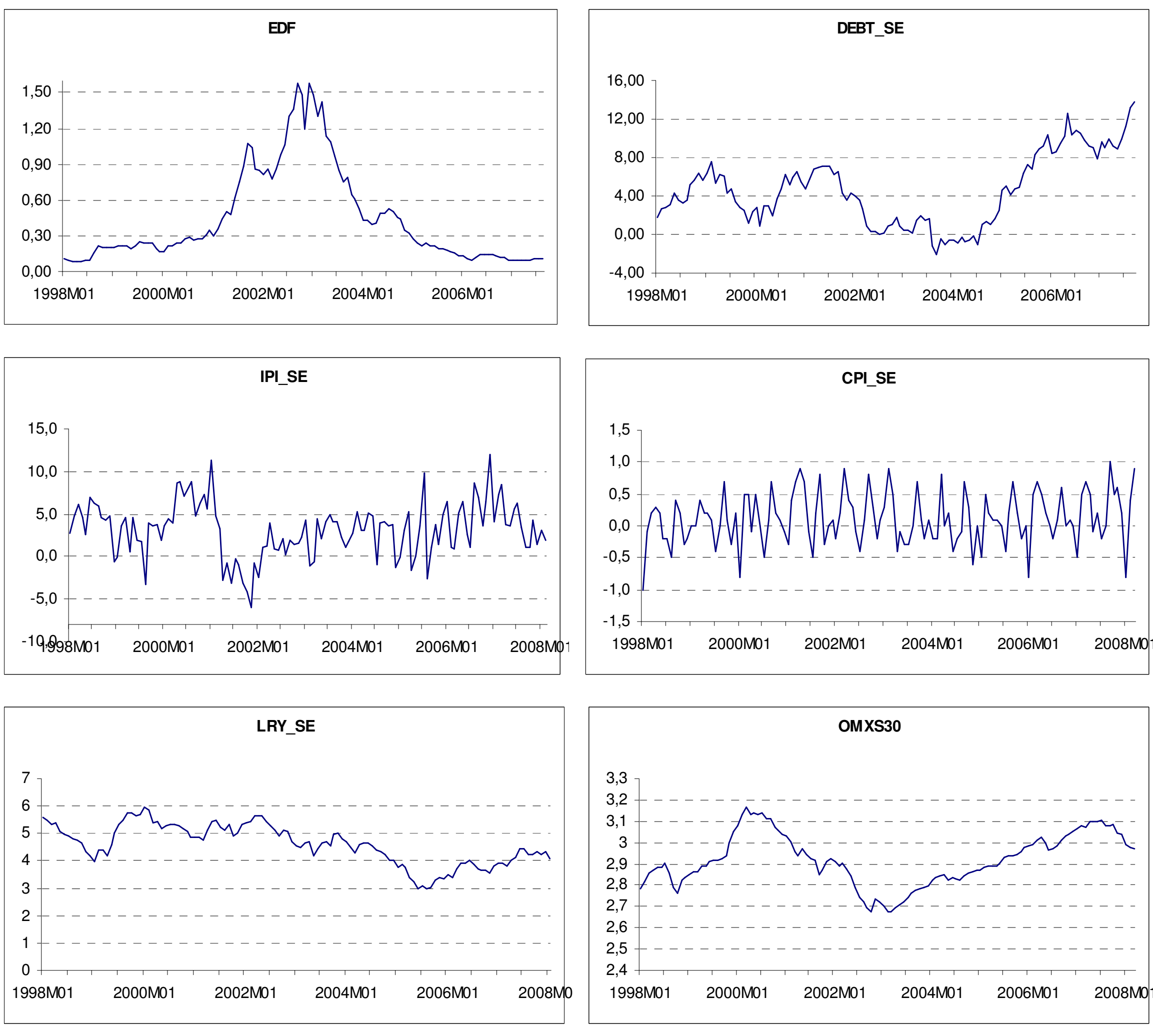

Figure 7 Monthly development of Swedish data

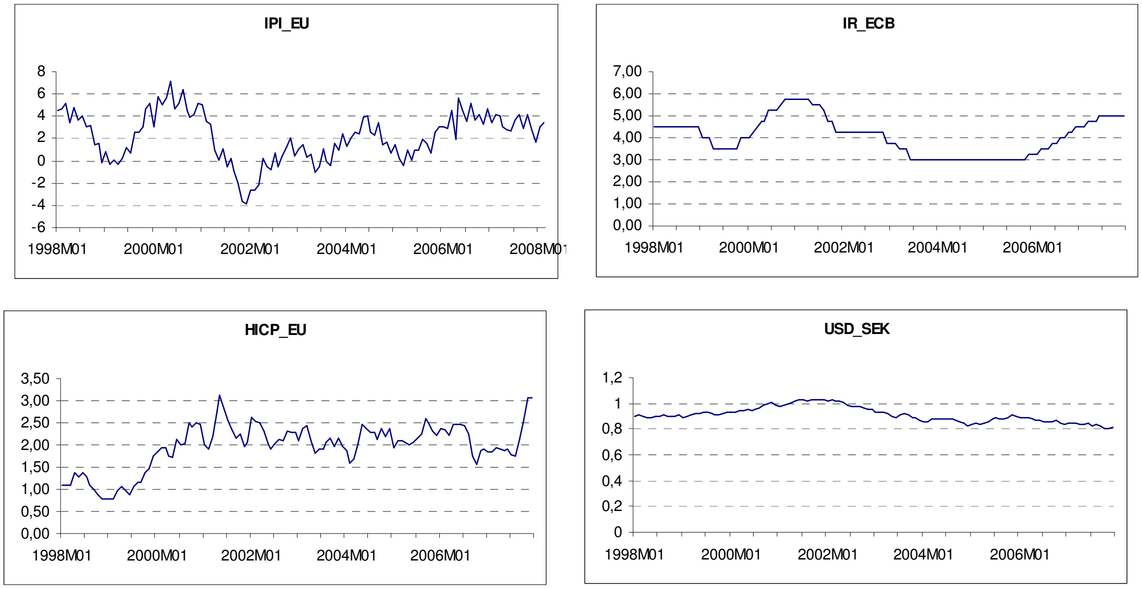

Figure 8 Monthly development of European data

From the graphs at Figure 7 and Figure 8 it is possible to notice that during the period of the high EDF, lending growth to companies (DEBT_SE) had decreasing trend and when EDF changed its direction and got decreasing trend, DEBT_SE had turned to the increasing direction. Inversed dependence it is also seen between EDF and the industry production change (both Swedish and European). At the same period from 2000 till 2004 OMXS30 index had negative trend either. The interest rate set by Europe central bank (IR_ECB) got negative trend just since 2001 second half. As a result, even just looking at the graphs it is obvious that macroeconomics can influence changes of EDF. Additionally, it is also seen than growing EDF entails different macroeconomic development.

Before implementing the methods that were put forward in the last chapter, several tests with empirical data have to be done. The most important test for time series is a test of stationary. It is possible to use several tests to check the stationary of series, as graphical (visible), autocorrelation (convenient use of correlograms), least variation analysis and Dickey Fuller or Augmented Dickey Fuller (ADF) tests.

The time series of Swedish and European data were tested just by using ADF. Test of stationary (ADF) in the level of the series and the first difference of the series were applied. As data is monthly, maximum lag was set up to 12 periods. Null hypothesis was that unit root exists.

Decision rule:

If	t* > ADF critical value, => not reject null hypothesis, i.e., unit root exists.
If	t* < ADF critical value, => reject null hypothesis, i.e., unit root does not exist.

The test indicated that all time series are integrated of order 1 (Appendix B).

Next Granger-causality test (with the lags from 3 months up to 2 years) was applied in order to learn which group of variables is useful to predict other variables. Clearly, the notion of Granger causality does not imply true causality, it only implies forecasting ability. Relations are presented in Appendix C in three cases: using just Swedish variables, just European variables and using all the variables.

In Appendix C the results of Granger-causality test using both Swedish and European empirical data (Appendix C, C.3) show that EDF can be described using USD_SEK and OMXS30 variables. In the next section, as the probability of default and its dependence from macro variables are the research object, it will be made a number of attempts to find how EDF can be modeled and tested by using macroeconomic data.

3.2 Modeling credit risk measure

In CreditPortfolioView (CPV) model the default rate p_t is usually measured as the ratio of non-performing loans[29] and total amount of the loans. There was a problem of getting non-performing loans in each industrial sector. It was one of the reasons not to run CPV with the empirical data. The aggregated cases for estimating expected default frequency (EDF) where borrowers of different sectors are not distinguished were analyzed. The main objective of an aggregate stress test is to help public authorities to identify structural vulnerabilities and overall risk exposures in a financial system that could lead to systemic problems. The most suitable models for financial stability estimation are VAR and VEC.

3.2.1. VAR and VEC models

All time series are non-stationary, so the first difference, as it makes the time series stationary (Appendix B), is used in VAR. While searching for a quite suitable model that

[29] A loan is nonperforming when payments of interest and principal are past due by 90 days or more, or at least 90 days of interest payments have been capitalized, refinanced or delayed by agreement, or payments are less than 90 days overdue, but there are other good reasons to doubt that payments will be made in full" (IMF)

estimates EDF by using macro variables, there were a number of the models with various macroeconomic factors and different number of lags constructed.

Firstly, it was run just with the Swedish data setting 12 lags. The results are given in the Appendix D, D.1. From all the variables EDF is described the best by applying the Swedish variables. Trying to find out which endogenous variables can be shifted as exogenous, it was reached that it makes not big difference without applying them at all and to leave just EDF dependent on its historical values (Appendix D, D.2).

The two most common methods for estimating the optimal lag length for a VAR, are the Akaike and Schwarz-Bayesian information criteria. In addition the usual diagnostic checks need to be made, to ensure the VAR is well specified. In particular the LM test for autocorrelation needs to be checked (the DW test can not be used with a VAR as it contains lagged dependent variables). If there is evidence of autocorrelation, more lags need to be added until the autocorrelation has been removed. At the beginning of constructing models it will be carried on with the set of 12 lags and the comparison of the models will be verified by AIC and SC criteria.

Using only the bunch of the European variables in making VAR model, the results are much better (AIC of the whole model is -2.6, when in the previous it was approx. 10) than just with the Swedish data (Appendix E, E.1). Then it was tried to move some variables (IPI_EU, HICP_EU and IR_ECB) from endogenous side to exogenous side and general result became better, AIC decreased down from -2.6 down to -7.6.

Finally, in this way of constructing the most suitable model it was tried to use both group of variables (Swedish and European) at the same time taking IPI_EU, HICP_EU and IR_ECB variables as exogenous (Appendix F, F.1). After some more variables shifting to exogenous side the result became the best in comparison with the previously constructed models (Appendix F, F.2).

Because of the good result (Appendix F, F.2, AIC and SC criteria) and according to the last created model (Appendix F, F.3), new groups of endogenous and exogenous variables were created. USD_SEK was moved to endogenous side, and DEBT_SE, IPI_SE and CPI_SE were moved to exogenous side (Table 3).

Table 3 Variables

PREVIOUS		NEW	
Endogenous (Swedish)	**Exogenous (European)**	**Endogenous**	**Exogenous**
EDF	IPI_EU	EDF	*DEBT_SE*
DEBT_SE	IR_ECB	LRY_SE	*IPI_SE*
IPI_SE	HICP_EU	OMXS30	*CPI_SE*
CPI_SE	USD_SEK	*USD_SEK*	IPI_EU
LRY_SE			IR_ECB
OMXS30			HICP_EU

For the next attempt it was a new group of endogenous variables (EDF, LRY_SE, OMXS30 and USD_SEK) cointegrated (Appendix G, G.1). Better cointegration is without LRY_SE (Appendix G, G.2). It was used 12 lags in the all previous cases, but in order to find out how many lags it has to be used, the Lag Length Criteria was run by Eviews (Appendix G, G.3).

Running Lag Length Criteria for EDF, OMXS30 and USD_SEK it was found out that the most informative lags are up to 4 (Appendix G, G.3). Then one more time VAR with 4 lags was run and for comparison cointegration with 4 lags was done either (Appendix H, H.1 and H.2).

Last attempt to model EDF was by including exogenous variables (Table 3, NEW) to VAR(4) and VEC(4) (it can be called as cointegration with 4 lags either), but the received results were even worse than without exogenous variables.

3.2.2. Comparison of the models

As it was mentioned before, to find the "best" model these criteria are used:

- likelihood ratio test criterion (LR);
- final prediction error criterion (FPE);
- Akaike information criterion (AIC);
- Scwarz information criterion (SIC);
- Hannan-Quin information criterion (HQ).

The "best" fitting model is the one that maximizes the LR, or minimizes the FPE criterion function or AIC, SIC or HQ. From the models that have been created (Appendices D-H) the lowest AIC and SIC are of the models VAR(4) and VEC(4) with variables EDF, OMXS30 and USD_SEK (Table 4). Actually, lag length selection was done considering above mentioned criteria.

Table 4 Comparison of the constructed models

Rating	Model	Variables	AIC	SC
BEST	VAR(4)	EDF, OMXS30, USD_SEK	-13	-12
BEST	VEC(4)	EDF, OMXS30, USD_SEK	-13	-12
GOOD	VEC(12)	EDF, OMXS30, USD_SEK	-11	-9
SATISFACTORY	VEC(12)	EDF, OMXS30, USD_SEK, LRY_SE	-10	-5
SATISFACTORY	VAR(12)	EDF, OMXS30, USD_SEK, LRY_SE	-10	-5

Additional requirement is that VAR residuals are not autocorrelated (and normal distributed). Thus the last steps were to check autocorrelation (Figure 9) and heteroskedasticity in residuals and if they are normally distributed (Appendix I, I.1-I.3).

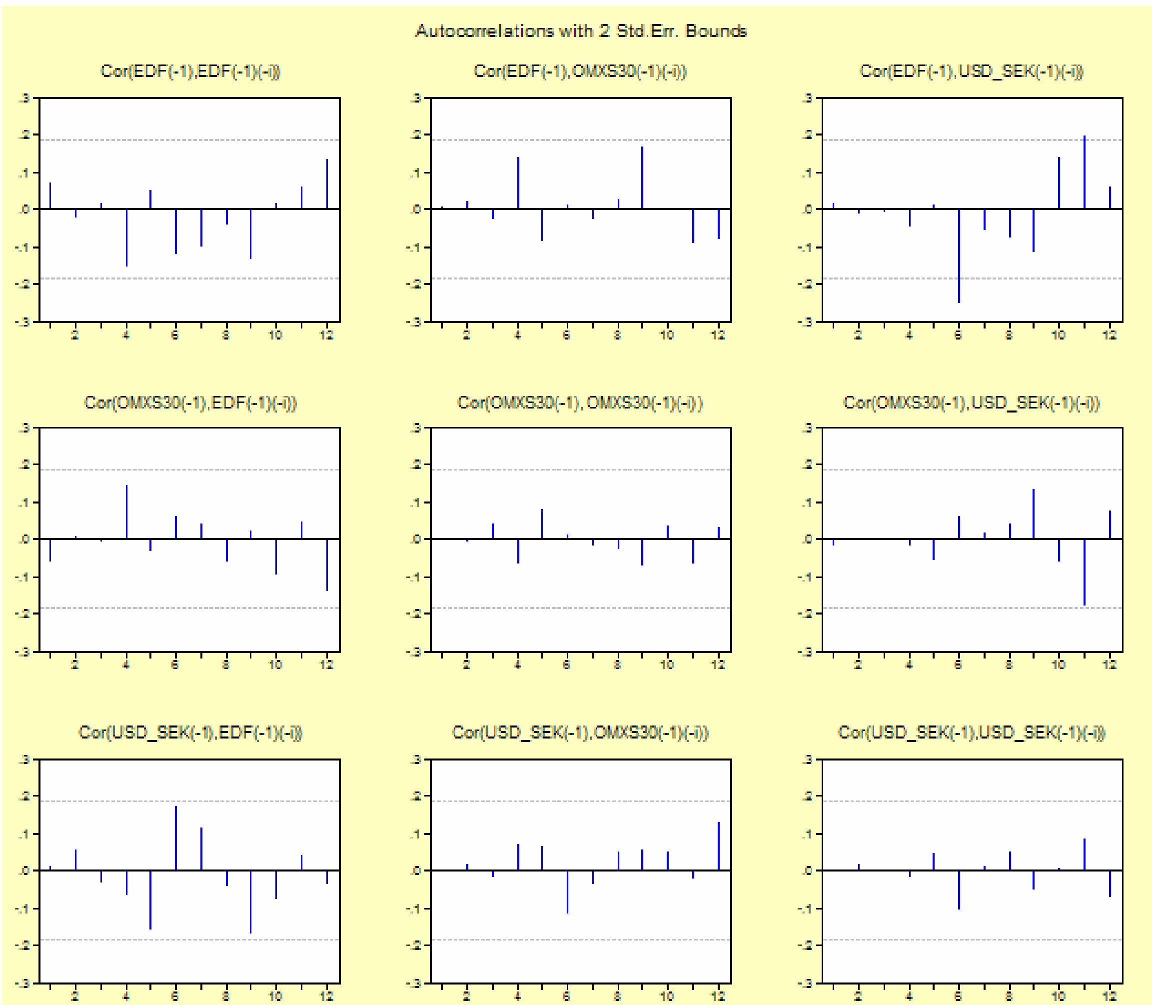

Figure 9 Autocorrelations

The same procedure with autocorrelation, heteroskedasticity, normality was done with the VEC(4) model. Not big changes were noticed.

Analyzing the results in Appendix I, it is might be possible to say that the residuals are not correlated, the probability was not stable among the different lags (Appendix I, I.1). The same can be said about heteroskedasticity, as the probability in some cases was really low, in the other even around 0.5 (Appendix I, I.2). Normality test results can be hardly summarized because the obstacle can be that there were not enough observations (Appendix I. I.3).

Residuals of both models (VAR(4) and VEC(4)) are given in Appendix J. It is seen that they are very similar the same as with AIC and SC criteria.

The representation of **VAR(4)** model is

EDF(-1) = **0.8219***EDF(-2) + **0.1400***EDF(-3) + **0.2992***EDF(-4) - **0.3471***EDF(-5) - **0.2056***OMXS30(-2) + **0.3632***OMXS30(-3) - **0.2239***OMXS30(-4) + **0.0491***OMXS30(-5) - **0.1941***USD_SEK(-2) + **0.6717***USD_SEK(-3) - **0.3401***USD_SEK(-4) + **0.4176***USD_SEK(-5) - **0.4196**

OMXS30(-1) = - **0.0370***EDF(-2) + **0.0461***EDF(-3) - **0.0034***EDF(-4) - **0.0108***EDF(-5) + **1.2471***OMXS30(-2) - **0.3103***OMXS30(-3) + **0.0892***OMXS30(-4) - **0.0718***OMXS30(-5) + **0.4240***USD_SEK(-2) - **0.6022***USD_SEK(-3) + **0.1458***USD_SEK(-4) - **0.0370***USD_SEK(-5) + **0.2004**

USD_SEK(-1) = - **0.0121***EDF(-2) + **0.0092***EDF(-3) + **0.0048***EDF(-4) - **0.0121***EDF(-5) + **0.0120***OMXS30(-2) + **0.0008***OMXS30(-3) - **0.0226***OMXS30(-4) + **0.0055***OMXS30(-5) + **1.2171***USD_SEK(-2) - **0.4447***USD_SEK(-3) + **0.2248***USD_SEK(-4) + **0.04***USD_SEK(-5) - **0.0180**

The representation of **VEC(4)** model is

D(EDF) = - **0.0971***(EDF(-1) + **0.5296***OMXS30(-1) - **5.0550***USD_SEK(-1) + **2.6300**) + **0.0112***D(EDF(-1)) + **0.0892***D(EDF(-2)) + **0.3225***D(EDF(-3)) - **0.2034***D(EDF(-4)) - **0.0662***D(OMXS30(-1)) + **0.1794***D(OMXS30(-2)) - **0.2991***D(OMXS30(-3)) - **0.0654***D(OMXS30(-4)) - **0.6123***D(USD_SEK(-1)) - **0.1250***D(USD_SEK(-2)) - **0.4000***D(USD_SEK(-3)) + **0.0206***D(USD_SEK(-4)) - 7.740800165e-05

D(OMXS30) = **0.008***(EDF(-1) + **0.5296***OMXS30(-1) - **5.055***USD_SEK(-1) + **2.6300**) - **0.0669***D(EDF(-1)) - **0.0043***D(EDF(-2)) + **0.0085***D(EDF(-3)) + **0.0473***D(EDF(-4)) + **0.2580***D(OMXS30(-1)) - **0.0250***D(OMXS30(-2)) + **0.1317***D(OMXS30(-3)) + **0.0281***D(OMXS30(-4)) + **0.4449***D(USD_SEK(-1)) - **0.1449***D(USD_SEK(-2)) + **0.0114***D(USD_SEK(-3)) - **0.0590***D(USD_SEK(-4)) + 0.0008

D(USD_SEK) = - **0.0093***(EDF(-1) + **0.5296***OMXS30(-1) - **5.055***USD_SEK(-1) + **2.63**) - **0.0023***D(EDF(-1)) + **0.0089***D(EDF(-2)) + **0.0101***D(EDF(-3)) + **0.0006***D(EDF(-4)) + **0.01330***D(OMXS30(-1)) + **0.0373***D(OMXS30(-2)) - **0.0209***D(OMXS30(-3)) + 0.0533*D(OMXS30(-4)) + **0.1663***D(USD_SEK(-1)) - **0.275***D(USD_SEK(-2)) - 0.**0590***D(USD_SEK(-3)) + **0.0294***D(USD_SEK(-4)) - 0.0009

The representations of the VAR(4) and VEC(4) models make out the equations of the used variables by describing them with historical values and the rest. Figure 10 shows one more time the dynamics of the variables that were used in the construction of the VAR(4) and VEC(4) models.

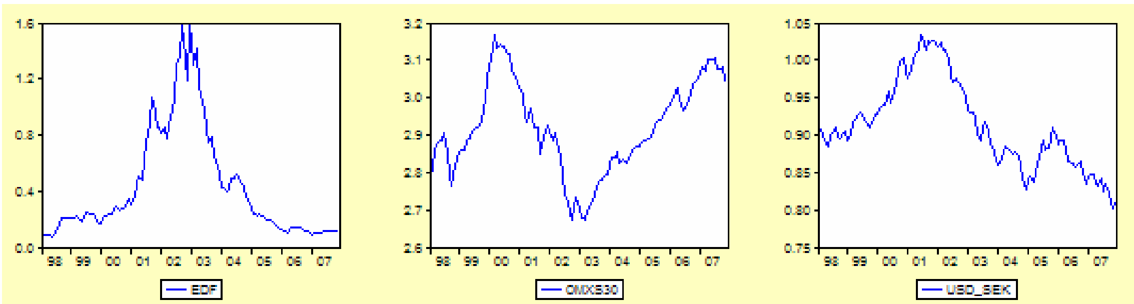

Figure 10 Variables

Moreover, the was done with Eviews Johansen Cointegration test for VEC(4), results are presented in Appendix K. The cointegration of variables EDF, OMXS30 and USD_SEK is represented in graphic (Figure 11).

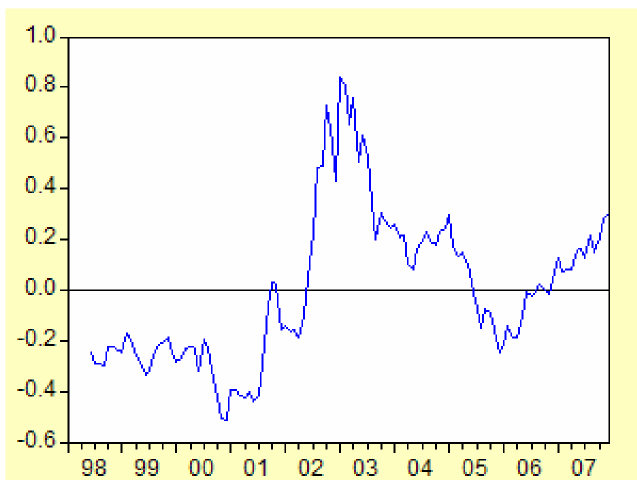

Figure 11 Cointegrating relation (EDF, OMXS30, USD_SEK)

At this moment the constructed models can be used for stress testing. As it was mentioned in previous sections, it is worth applying macroeconomic scenarios and sensitivity analysis by each variable.

3.3 Macroeconomic impact

After creating macroeconomic scenario it needs to translate the scenario into stress of systematic factors of a credit risk model. Systematic risk factors must have a clear economic interpretation. The economic stress scenario is translated into constraints on the corresponding systematic factors. These constraints are used to truncate the distribution of the stressed risk factors or in other words – restrict the state space of the model. The stress scenarios are chosen in such a way that the translation involves only a small number of systematic factors. The response of the peripheral (or unstressed) risk factors is specified by the dependence structure of the model. Using the made-up model the macroeconomic stress testing can be run.

3.3.1. Sensitivity analysis

More simply, stress testing is a way to produce alternative scenarios applying sensitivity analysis. Sensitivity analysis is the study of how the variation (uncertainty) in the output of a mathematical model can be apportioned, qualitatively or quantitatively, to different sources of variation in the input of a model. Impulse response functions will be used in order to run sensitivity analysis for VAR and VEC models.

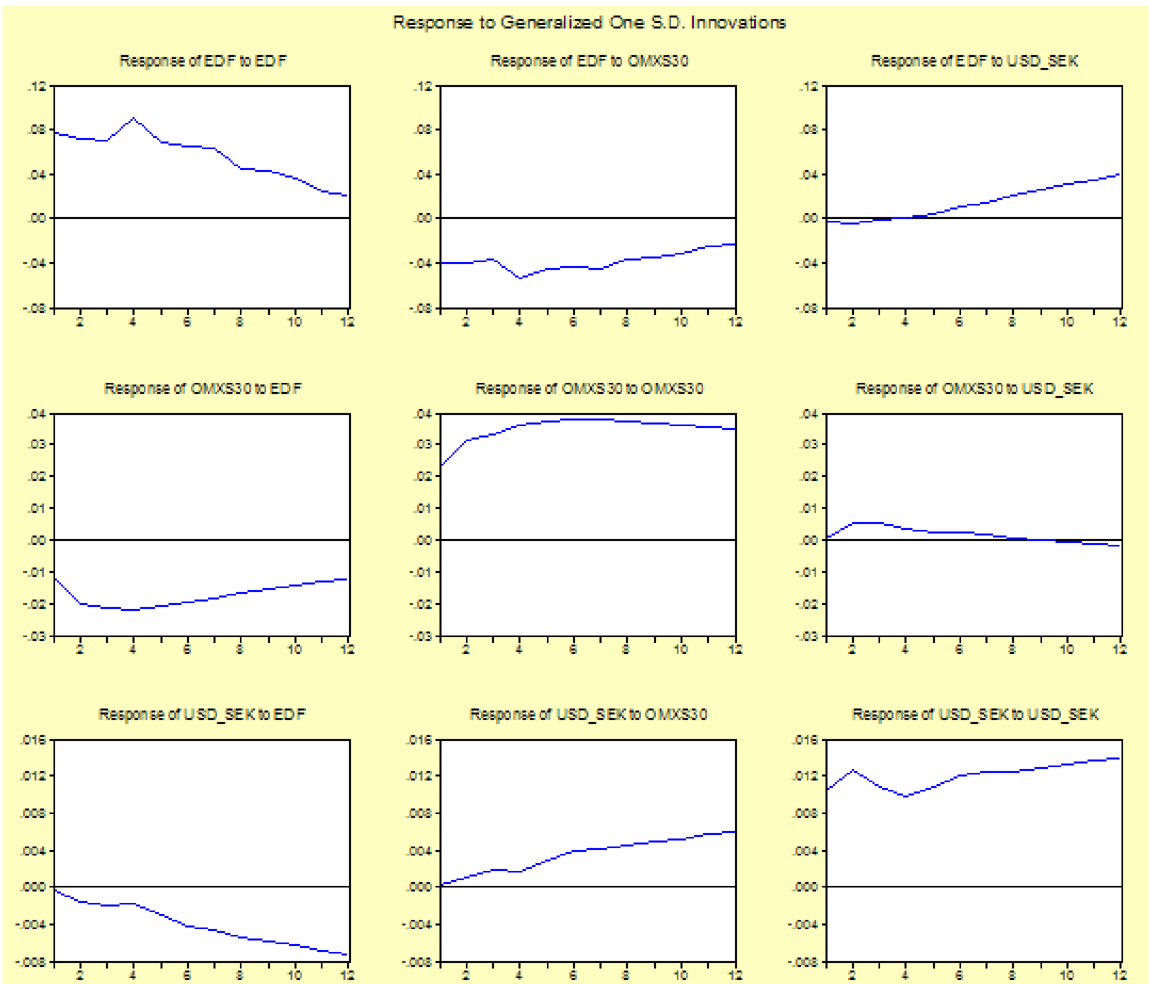

Figure 12 Impulses of response in VEC(4)

The graphical analysis is able to make by looking at Figure 12. Impulses of response in VAR(4) are presented in Appendix L, L.1. In order to understand trends of the each variable it is better to analyze cumulative graphics (Figure 13).

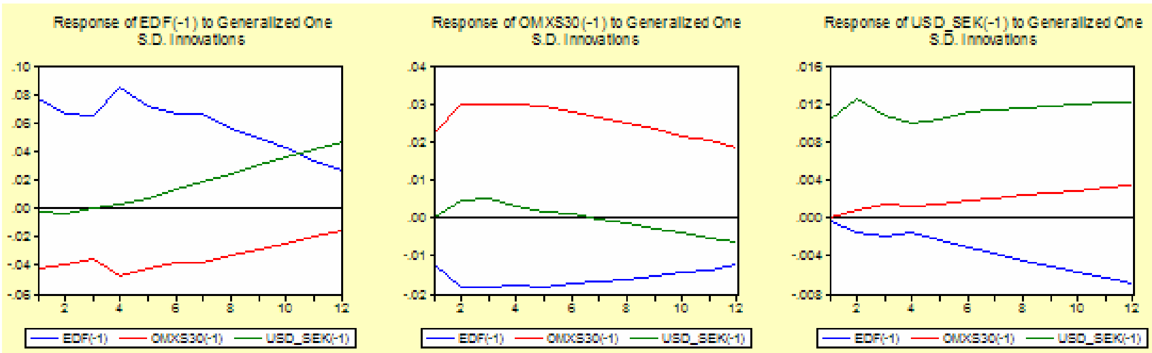

Figure 13 Impulse of response in VEC(4), combined

It is visible that increasing OMXS30 and USD_SEK make EDF go down. Decreasing USD_SEK and increasing EDF, force OMXS30 to decrease. And quite strong decrease of EDF and increase of OMXS30 gives to USD_SEK slightly growing trend.

3.3.2. Scenario analysis

As it is known from the previous sections, scenario analysis is when several factors are changed in stress testing. In order to find out which variables and how they have to be changed it is needed to analyze the history of variables (Table 5). Moreover, for scenario analysis it is planned to use some exogenous variables chosen at random.

.

Table 5 Some characteristics of variables

	EDF	OMXS30	USD_SEK	DEBT_SE	HICP_EU	IR_ECB	IPI_EU
Minimum	0,07789	2,6723	0,8007171	-2,0228	0,7857	3	-3,9
Maximum	1,58126	3,1668	1,0324294	13,8235	3,2682	5,75	7,1
Mean	0,42691	2,9186	0,91134	4,5706	1,9835	4,0625	2,1409
Standard deviation	0,39394	0,1231	0,0595	3,6775	0,5436	0,8570	2,1797

Usually values fluctuate in 3σ interval, thus to calculate the values that can have significant change, it needs to find 3σ values and for stress testing to use values exceeding these boundaries (Table 6).

Table 6 Values exceeding three standard deviations

	EDF	OMXS30	USD_SEK	DEBT_SE	HICP_EU	IR_ECB	IPI_EU
3σ	1,19	0,37	0,18	11,04	1,65	2,6	6,54

To find extreme values, the values of 3σ were added or subtracted to/from the mean values.

Table 7 Extreme values

	EDF	OMXS30	USD_SEK	DEBT_SE	HICP_EU	IR_ECB	IPI_EU
Mean +/- 3σ	1,62	2,55	1,09	-6,47	3,63	6,66	-4,40

All the values in Table 7 are exceeding minimum or maximum values. It means that those values are extreme in comparison with ten years history and can be used for stress testing.

The main thing that has to be concerned is adverse trend of each variable (Table 8). Additionally, it is important to estimate the trends of couple variables at the same time. For example, it OMXS30 is increasing, usually IPI_EU is increasing either.

Table 8 Adverse trends

Notation	Adverse trend
OMXS30	decreasing
DEBT_SE	decreasing
USD_SEK	increasing
HICP_EU	increasing
IR_ECB	increasing
IPI_EU	decreasing

VAR(4) with the exogenous variables are represented in Appendix L. Although for the transparent example of scenario analysis, the equation of EDF(-1) is going to be used for simplified scenario analysis (Table 9).

Table 9 Scenario equation[30]

EDF(-1) = 0.9003*EDF(-2) - 0.1644*OMXS30(-1) + 0.5726*USD_SEK(-1)

As it is possible to notice that from the equation it was needed to remove the other variables that were insignificant and there left just two of them. Looking back to the Appendix C, C.3, the equation in Table 9 is a proof of Granger Causality Test.

[30] The equation was constructed using least squares method, results are given in Appendix L, L.3-4.

Scenario analysis results are presented in Table 10. However, the results of EDF_test have not exceeded historical EDF value's boundary. It means that there is no significant impact of macroeconomic factors to credit risk metric.

Table 10 Scenario analysis (3σ)

	OMXS30	USD_SE	EDF_test
Mean	*2,9186*	*0,91134*	*0,4269*
OMXS_30	**2,55**	0,91134	**0,4870**
USD_SE	2,9186	**1,09**	**0,5287**
Scenario	**2,55**	**1,09**	**0,5893**

For this reason it was tried to stress more macroeconomic variables up to 5σ difference from the mean value (Table 11). However, this adverse condition has not influenced EDF either as it would have exceeded the historical values.

Table 11 Scenario analysis (5σ)

	OMXS30	USD_SE	EDF_test
Mean	*2,9186*	*0,91134*	*0,4269*
OMXS_30	**2,30**	0,91134	**0,5281**
USD_SE	2,9186	**1,21**	**0,5974**
Scenario	**2,30**	**1,21**	**0,6991**

3.3.3. Results analysis

Summarizing the empirical results of credit risk stress testing it might be not right to state the macroeconomic variables can not change the credit risk metric significantly. One reason for the weak relationship found between EDFs and macrofundamentals could be that much of the business cycle volatility in default probabilities has already been smoothed out in the construction of EDFs.[31] For this reason as a credit risk metric it would be worth using any other probability of default instead of EDF. It might be substituted by the ratio of non-performing loans and total amount of the loans for corporate clients. This ratio would be the historical probability, not as EDF that is a forward-looking measure. Anyway, the empirical analysis allowed estimating EDF by applying macroeconomic factors into VAR and VEC models that described the macroeconomic environment in the most suitable way because of the

[31] Sorge M., Virolainen K. (2005) A comparative analysis of macro stress-testing methodologies with application to Finland. p. 30;

ability to have two-way influence to the variables. The constructed models are suitable for sensitivity analysis by finding the response of the each variable. It was more difficult to use the latter models for scenario analysis as in the models there are a lot of coefficients that are complicated to interpret. Therefore, the scenario analysis was run by using the unsophisticated regression model in order to show how scenario analysis can be carried out. For future research it would be worth exploring how to simplify vector models and to use them for scenario analysis that even the person that has no high skills in statistics or mathematics would be able to understand the process of scenario analysis.

4. Publication

The part of the models presented in this project was published after 11th Lithuanian young scientists' conference "Science – future of Lithuania", occurred March 27th, 2008 in Vilnius.

MATEMATIKA
11-osios Lietuvos jaunųjų mokslininkų konferencijos „Mokslas – Lietuvos ateitis“, įvykusios Vilniuje 2008 m. kovo 27 d., medžiaga

KREDITO RIZIKOS VERTINIMAS TESTUOJANT NEPALANKIOMIS SĄLYGOMIS

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Anotacija. Nagrinėjamas kredito rizikos vertinimas testuojant nepalankiomis sąlygomis. Pateikiamas dažniausiai pasaulio bankų praktikoje naudojamas CreditPortfolioView makreoekonometrinis modelis, aprašomi veiksmai, kaip reikėtų šį modelį naudoti testuojant nepalankiomis sąlygomis. Taip pat yra aprašomas Merton modelis, kuris gali būti naudojamas atliekant šį kredito rizikos vertinimą. Pabaigoje yra pateikiamos nuorodos tolesnei šių modelių analizei.

Įvadas

Lietuvoje vis dar pagrindinė bankų veikla yra kreditavimas, dėl to bankams kredito rizika yra reikšmingiausia iš visos finansinės rizikos. Paprastai kredito rizikos vertinimas atliekamas normaliomis sąlygomis, bet visapusiškai kredito rizikos valdyti reikia taip pat įvertinti kredito riziką esant nepalankioms sąlygoms, nes rizikos veiksniai gali turėti skirtingą įtaką kredito rizikai esant kitokioms sąlygoms. Rizikos veiksnių įtaką esant normalioms sąlygoms galima prognozuoti remiantis praeitimi, bet rizikos veiksnių įtaka esant nepalankioms sąlygoms yra sunkiai nuspėjama.

Atsižvelgiant į pasaulio bankų patirtį, šiame straipsnyje išsamiau bus aptariamas CreditPortflioView modelis ir trumpai aprašomas Merton modelis.

Testavimas nepalankiomis sąlygomis

Testavimas nepalankiausiomis sąlygomis (angl. stress-testing) – tai rizikos valdymo priemonė (technika), naudojama vertinant galimą įtaką banko finansinei būklei, įvykus tam tikram (apibrėžtam) įvykiui (įvykiams) ir (ar) pakitus (esant nepastoviai) finansinei ar ekonominei aplinkai.[32]

Testavimas nepalankiomis sąlygomis leidžia iš anksto įvertinti kredito riziką, kuri tikriausiai padidėtų, jei staiga iš esmės pasikeistų rinkos sąlygos neigiama linkme. Taip pat, įvertinus galimus pokyčius, galima iš anksto pasirengti, kaip nuo tokių pokyčių apsisaugoti. Dažniausiai rinkos sukrėtimai teikia nemažai nuostolių finansinėms institucijoms. Testavimas nepalankiomis sąlygomis yra galimų ateities įvykių modeliavimas. Galimi ateities įvykiai gali būti ne tik makroekonominių sąlygų pasikeitimas, bet ir kitokie įvykiai, kurie galėtų pakeisti kredito portfelio riziką.

Kredito rizikai testuoti nepalankiomis sąlygomis yra naudojami šie modeliai: CreditPorftolioView modelis (Wilson[33] 1997) ir Merton (1974) modelis.

[32] (2007) Testavimo nepalankiausiomis sąlygomis bendrosios nuostatos. *Lietuvos bankas*, p. 1;
[33] Wilson (1997) vienas iš pirmųjų pasiūlė kredito rizikos vertinimo modelį, kuris tiksliai susisieja su makroekonominiais veiksniais ir ūkio sektorių įsipareigojimų neįvykdymo tikimybe.

Makroekonometrinis modelis

CreditPortfolioView modelis yra logarimtinė regresija, kuri aprašo, kaip įsipareigojimų neįvykdymo lygis priklauso nuo makroekonominių veiksnių kiekvienam ekonominės veiklos sektoriui. Paprastai yra naudojami tokie makroekonominiai veiksniai kaip realusis BVP, nominali palūkanų norma ir/ar kiekvieno sektoriaus įsiskolinimo rodikliai. Įsipareigojimų neįvykdymo lygis yra bankrutavusių įmonių ir visų įmonių santykis kiekviename ūkio sektoriuje atskirai.

Modelį sudaro dvi pagrindinės sudedomosios dalys:

- empirinių lygčių sistema;
- Monte Carlo imitacija galimoms įsipareigojimų neįvykdimo tikimybėms (arba kredito nuostoliams) nustatyti.

Empirinių lygčių sistema

Vidutinis sektoriaus įsipareigojimų neįvykdymo lygis (PD, angl. probability of default) yra modeliuojamas naudojant šią formulę

$$p_{j,t} = \frac{1}{1+e^{y_{j,t}}},$$

čia j yra ekonominės veiklos sektoriai, kuriems bankas skolina pinigus, $p_{j,t}$ yra sektoriaus j (j = 1,..., J) vidutinis įsipareigojimų neįvykdymo lygis laikotarpiu t¸ o $y_{j,t}$ yra ūkio sektoriaus specifinis makroekonominis indeksas. Kadangi $p_{j,t}$ turi būti nuo nulio ir vieneto, naudojama logistinė transformacija

$$y_{j,t} = \ln\left(\frac{1-p_{j,t}}{p_{j,t}}\right).$$

Modelyje yra analizuojama $y_t = (y_{1,t},..., y_{J,t})$‘ tiesinė priklausomybė nuo makroekonominių veiksnių, ankstesnių ūkio sektorių makroekonominio indeksų.

Pagrindinė sistemos lygtis yra

$$y_t = m + A_1 x_t + ... + A_{1+s} x_{t-s} + \phi_1 y_{t-1} + ... + \phi_k y_{t-k} + v_t, \quad (1)$$

čia x_t yra $M \times 1$ makroekonominių rodiklių vektorius, m yra $J \times 1$ laisvų narių vektorius; $A_1,\ldots, A_{1+s}$ yra $J \times M$ ir $\phi_1,...,\phi_k$ yra $J \times J$ koeficientų matricos; ir v_t yra $J \times 1$ paklaidų vektorius.

Kita sistemos lygtis yra M makroekonominių kintamųjų modeliavimas. Pagal Wilson (1997), tai turėtų būti autoregresijos modelis. Bendruoju atveju lygtis yra

$$x_t = n + B_1 x_{t-1} + B_p x_{t-p} + \Theta_1 y_{t-1} + ... + \Theta_q y_{t-q} + \varepsilon_t, \quad (2)$$

čia n yra $M \times 1$ laisvų narių vektorius; $B_1,\ldots, B_p$ yra $M \times M$ *ir* $\Theta_1,...,\Theta_q$ yra $M \times J$ koeficientų matrica; ir ε_t yra $M \times 1$ paklaidų vektorius.

Paklaidos v_t ir ε_t yra koreliuotos. Paklaidų struktūra atrodo taip:

$$e_t = \begin{pmatrix} v_t \\ \varepsilon_t \end{pmatrix} \sim N(0, \Sigma), \quad \Sigma = \begin{pmatrix} \sum_v & \sum_{v,\varepsilon} \\ \sum_{v,\varepsilon} & \sum_\varepsilon \end{pmatrix}. \qquad (3)$$

Monte Carlo imitacija

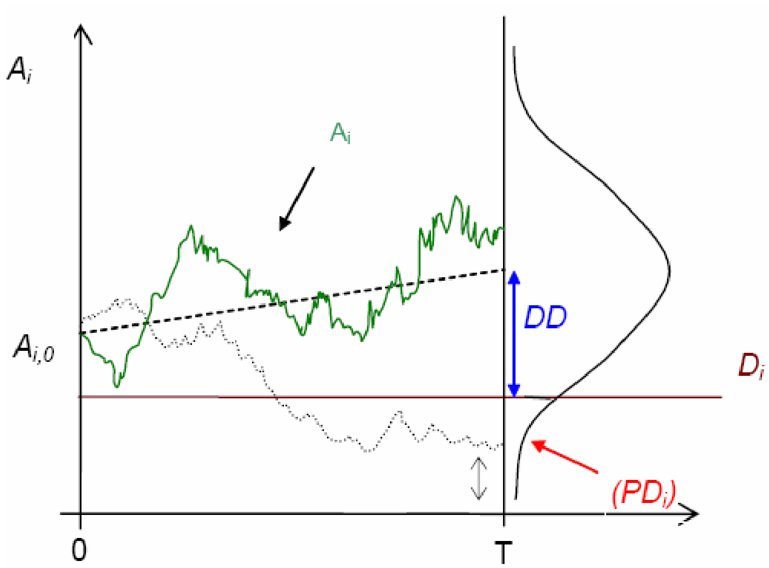
Įvertinus parametrus, kreditų portfelio nuostolio skirstiniui įvertinti yra naudojamas Monte Carlo metodas. Šis metodas leidžia nesunkiai įvertinti kreditų rizikos koreliaciją, matematiškai nėra sudėtingas, nors patys skaičiavimai ir trunka palyginti ilgai. Modeliuojant Monte Carlo metodu, daug kartų generuojamos tiesiogiai nestebimo paskolų rizikos veiksnio x atsitiktinės reikšmės, randami šias reikšmes atitinkantys PD rodikliai, radus PD rodiklius, tarp kurių yra koreliacija, skaičiuojama paskolų vertė, o susumavus paskolų vertes randama viso paskolų portfelio vertė. Tokius veiksmus atlikus daug kartų, randamos įvairių kreditų portfelių vertės, o iš jų sudaromas paskolų portfelio nuostolio skirstinys (Valvonis 2006). Šis metodas apskritai yra naudojamas ekonominiam kapitalui apskaičiuoti.

Norint atlikti testavimą nepalankiomis sąlygomis naudojant CreditPortfolioView modelį, pirmiausia pasirenkamas veiksnys, pavyzdžiui, esant 4 % palūkanų normos padidėjimui, t. y. atliekama jautrumo analizė palūkanų normos didėjimui. Kitas būdas yra scenarijaus sukūrimas, t. y. daugiau negu vieno veiksnio padidinimas ir/ar sumažinimas, atsižvelgiant į jų dispersiją (blogiausiam scenarijui vertinti tikslingiausia būtų veiksnį pakeisti didesne procentine dalimi nei yra to veiksnio dispersija).

Merton modelis

Merton modelis yra klasikinis kredito rizikos modelis. Listinguojamos bendrovės įsipareigojimų neįvykdymo tikimybė gali būti modeliuojama Merton tipo barjeriniu opciono būdu.

Grafiškai modelis pavaizduotas paveiksle (Drehman 2005).



i-osios bendrovės vertės A_i kaita yra stochastinis procesas su tikėtina grąža μ_i ir sklaida σ_i, i = 1,…n,

$$dA = \mu_i A_i dt + \sigma_i A_i dz, \qquad (4)$$

čia z yra Brauno judesys. Kai turto vertė A_i yra mažesnė nei įsipareigojimų neįvykdymo lygis D_i, tuomet bendrovė i bankrutuoja. Dažnai įsipareigojimų neįvykdymo tikimybė *PD* yra tapatinama su laikotarpiu iki įsipareigojimų neįvykdymo ir žymima *DD*.

Testavimas nepalankiomis sąlygomis naudojant Merton modelį

Skaičiavimams yra naudojama europinio pirkimo pasirinkimo sandorio vertė

$$S_t = A_t N(d_1(A_t, T-t)) - De^{-r(T-t)} N(d_2(A_t, T-t)), \qquad (5)$$

čia $d_1(a,t) = \dfrac{\log\dfrac{a}{D} + (r + \dfrac{\sigma_A^2}{2})t}{\sigma_A \sqrt{t}}$,

$$d_2(a,t) = d_1(a,t) - \sigma_A \sqrt{t}, \quad N(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-\frac{1}{2}u^2} du,$$

S_t yra bendrovės kapitalo vertė, r – obligacijos ar banko sąskaitos palūkanų norma.

Testuojant nepalankiomis sąlygomis, yra pakeičiamos S_0, r arba σ_A vertės naujomis.

$$\sigma_A^{nauja} = \sigma_A^{sena}(1 + \Delta\sigma_A). \qquad (6)$$

Dėl bendrovės kapitalo vertės ir palūkanų pakeitimo siūloma palūkanų norma palikti nepakeistą ir keisti tik kapitalo vertę.

$$S_0^{nauja} = S_0^{sena}(1 + \Delta S)e^{-\Delta rT}. \qquad (7)$$

Iš (7) lygties matyti: jei yra didinama palūkanų norma, tuomet kapitalo vertė taip pat padidėja, ir atvirkščiai. Dėl šios priežasties (7) yra $e^{-\Delta rT}$ daugiklis.

Išvados

1. CreditPortfolioView modelis leidžia rizikos problemą nagrinėti išsamiau, nes analizuojama ne tik kredito, bet rinkos rizika.

2. CreditPortfolioView modelis yra patogus naudoti, nes nesunkiai galima rasti makroekonominių veiksnių įtaką įsipareigojimų neįvykdymo lygiui.

3. Makreoekonometriniame CreditPortfolioView modelyje yra pateikta makroekonominio indekso tiesinė priklausomybė nuo makroekonominių veiksnių, tačiau modelį reikėtų pakoreguoti, nes indeksas gali priklausyti ir netiesiškai.

4. Merton modelis gali būti naudojamas tik įmonėms, kurios yra listinguojamos, nes modeliui reikia rinkos kapitalo vertės.

5. Merton modelio trūkumas yra tas, kad paprastai bendrovės įsipareigojimai yra komplikuotesni nei įsipareigojimų lygis D su terminu T. Bendrovės dažnai turi keletą skirtingų terminų kreditų. Taip pat įmonė nebūtinai turi bankrutuoti, jei jos turto vertės lygis tampa mažesnis nei įsipareigojimų lygis.

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STRESS TESTING IN CREDIT RISK ANALYSIS

Giedrė Ramanauskaitė

Summary

The study deals with the stress testing in credit risk analysis. I have discussed one of the most popularly used models in world banks and have written the procedure used for macroeconomic credit risk stress testing. Moreover, there is shortly overviewed another model that can be used for credit stress testing. Finally, some directions for future research are given as well.

5. Conclusions

While analyzing literature related to credit risk assessment and measurement, conclusion was made that stress testing is the part of any financial risk analysis that is rather interesting for scientists and risk analysts. For this reason the aim to explore which statistical and mathematical methods are suitable for credit risk stress testing was developed for the master project.

I have explored what kind of models can be used for credit risk stress testing. Moreover, I have investigated how these models include macroeconomic impact to credit risk, i.e. a probability of default. In the class of multivariate linear models, pure VARs are currently dominating in macroeconomic applications. This model is extremely appropriate for economical connections specification. Furthermore, because of the impulse response functions it involves the shock analysis, when it is estimated reactions of each variable, at the moment when one of the system's variables is stressed. Unfortunately, VAR models may require a rather large lag length in order to describe a series adequately. As the time series of macroeconomic variables usually are non-stationary VEC model can be used either. However, these models have disadvantages either. When any of the vector models is constructed it usually contains a lot of terms and every term has its coefficient. When there are a number of coefficients it becomes too complicated to interpret them.

The commonly used model in general credit risk analysis is based on Merton and it is called Moody's KMV model. The model contains the total value of company's assets that fluctuates and it is the reason, why company is capable to default. The value of the assets is not difficult to estimate by using option pricing theory. At the same time this model might be more suitable for general stress testing by decreased assets and increased liabilities than for macroeconomic stress testing in credit risk analysis by including macro variables with impact to assets and liabilities.

Most such kind of models so far have focused on credit risk only, usually limited to a short-term horizon. Banks come with more financial risks than credit risk and macroeconomic shocks can have longer consequences than one year. Hence it would be desirable, therefore, to lengthen the horizon of macro stress-tests (so far typically limited to one year) allowing for serially correlated shocks to build up economic imbalances over time.

Empirical analysis indicated that the straightforward models fit the best. It was tried to add a lot of macroeconomic variables while constructing model, but the evaluation of the models were the finest when there were just several factors included. As a result, it summarizes that stress testing can be started from the simple constructions. There is no need to start with complicated models. Moreover, the empirical analysis was made applying the aggregated credit risk measure. To determine more accurate results, it might be good to run credit risk stress testing for each sector when the probability of default is distinguished for each industrial sector separately. CreditPortfolioView is the model that is used the most popularly in the world banks, but to run empirical analysis for the implementing of this model intervened the restriction of the credit risk data.

In particular, macro stress-testing needs to pay closer attention to the correlation of risks and of risk measures over time and across institutions, to the length of the time horizon used for simulations and to the potential instability of all reduced-form parameter estimates because of feedback effects. Looking only at the aggregate "macro portfolio" of the banking sector, ignoring inter-bank exposures, might lead to underestimating the overall risk in the system.

In general, while substantial progress has been made in the last five years in developing quantitative techniques that help assess the vulnerability of financial systems, a number of methodological challenges still remain for future research.

The questions that should be answered for forthcoming research:

- Are macroeconomic credit risk stress testing methods reliable?
- How can their accuracy be controlled and improved?
- How can these methods be implemented efficiently?

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Appendix A

A.1 Descriptive statistics

	EDF	*IPI_SE*	*CPI_SE*	*LRY_SE*	*DEBT_SE*	*OMXS30*	*IPI_EU*	*IR_ECB*	*HICP_EU*	*USD_SEK*
Mean	0,42691	3,1485	0,1074	4,5964	4,5706	2,9186	2,1409836	4,0625	1,98352	0,9113406
Standard Error	0,03596	0,2899	0,0368	0,0660	0,3400	0,0111	0,1973396	0,0782361	0,04921	0,0054179
Median	0,23684	3,5708	0,1000	4,6550	4,3138	2,9084	2,4	4,25	2,05735	0,9012676
Mode	0,11	#N/A	0,2000	5,1350	#N/A	#N/A	3,1	3	0,788	0,8934455
Standard Deviation	0,39394	3,2019	0,4068	0,7256	3,6775	0,1231	2,1796866	0,8570334	0,54358	0,0595971
Sample Variance	0,15519	10,2522	0,1655	0,5264	13,5239	0,0152	4,7510337	0,7345063	0,29548	0,0035518
Kurtosis	0,97102	0,3580	-0,1873	-0,7266	-0,6977	-0,7432	-0,1920318	-0,9431693	0,10001	-0,6660211
Skewness	1,39994	-0,0661	-0,0594	-0,2925	0,3261	0,0080	-0,3100089	0,2938375	-0,40532	0,4107437
Range	1,50337	18,0680	2,0000	2,9750	15,8462	0,4945	11	2,75	2,4825	0,2317123
Minimum	0,07789	-6,0190	-1,0000	2,9950	-2,0228	2,6723	-3,9	3	0,7857	0,8007171
Maximum	1,58126	12,0490	1,0000	5,9700	13,8235	3,1668	7,1	5,75	3,2682	1,0324294
Sum	51,2298	384,1147	13,1000	556,1620	534,7659	356,0660	261,2	487,5	241,989	110,27222
Count	120	120	120	120	120	120	120	120	120	120

A.2 Correlation matrices

Endogenous	*IPI_SE*	*CPI_SE*	*LRY_SE*	*DEBT_SE*	*OMXS30*
IPI_SE	1				
CPI_SE	-0,106601646	1			
LRY_SE	-0,101191882	-0,022351635	1		
DEBT_SE	0,143134103	0,103936443	-0,495696529	1	
OMXS30	0,354351007	0,090476845	-0,060369757	0,608642505	1

Exogenous	*IPI_EU*	*IR_ECB*	*HICP_EU*	*USD_SEK*
IPI_EU	1			
IR_ECB	0,2616939	1		
HICP_EU	-0,0730158	0,0658249	1	
USD_SEK	-0,3409589	0,4335656	0,1041769	1

Appendix B

EDF		*level* t-Statistic	Prob.*	*1st difference* t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-1.893232	0.3345	-3.876025	0.0030
Test critical values:	1% level	-3.487550		-3.487550	
	5% level	-2.886509		-2.886509	
	10% level	-2.580163		-2.580163	

DEB_SE		*level* t-Statistic	Prob.*	*1st difference* t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-0.590238	0.8675	-11.76868	0.0000
Test critical values:	1% level	-3.486064		-3.486551	
	5% level	-2.885863		-2.886074	
	10% level	-2.579818		-2.579931	

IPI_SE		*level* t-Statistic	Prob.*	*1st difference* t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-3.283512	0.0179	-10.60510	0.0000
Test critical values:	1% level	-3.487046		-3.487550	
	5% level	-2.886290		-2.886509	
	10% level	-2.580046		-2.580163	

CPI_SE		*level* t-Statistic	Prob.*	*1st difference* t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-1.895187	0.3335	-8.101859	0.0000
Test critical values:	1% level	-3.491928		-3.491928	
	5% level	-2.888411		-2.888411	
	10% level	-2.581176		-2.581176	

LRY_SE		*level* t-Statistic	Prob.*	*1st difference* t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-1.761157	0.3981	-9.484777	0.0000
Test critical values:	1% level	-3.486064		-3.486551	
	5% level	-2.885863		-2.886074	
	10% level	-2.579818		-2.579931	

OMXS30		*level* t-Statistic	Prob.*	*1st difference* t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-1.532742	0.5137	-7.288819	0.0000
Test critical values:	1% level	-3.486551		-3.486551	
	5% level	-2.886074		-2.886074	
	10% level	-2.579931		-2.579931	

IPI_EU		*level* t-Statistic	Prob.*	*1st difference* t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-2.543714	0.1079	-5.440059	0.0000
Test critical values:	1% level	-3.487550		-3.487550	
	5% level	-2.886509		-2.886509	
	10% level	-2.580163		-2.580163	

IR_ECB		*level* t-Statistic	Prob.*	*1st difference* t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-2.559172	0.1045	-3.245381	0.0199
Test critical values:	1% level	-3.488063		-3.487550	
	5% level	-2.886732		-2.886509	
	10% level	-2.580281		-2.580163	

HICP_EU		*level* t-Statistic	Prob.*	*1st difference* t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-2.553544	0.1057	-9.139323	0.0000
Test critical values:	1% level	-3.486551		-3.486551	
	5% level	-2.886074		-2.886074	
	10% level	-2.579931		-2.579931	

USD_SEK		*level* t-Statistic	Prob.*	*1st difference* t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-0.658304	0.8520	-8.552718	0.0000
Test critical values:	1% level	-3.486551		-3.486551	
	5% level	-2.886074		-2.886074	
	10% level	-2.579931		-2.579931	

Appendix C

C.1 Swedish

Dependent	X_1 (t-lag)	X_2 (t-lag)
IPI_SE	$\text{OMXS30}_{t-6,\ t-12,\ t-24}$	$\text{EDF}_{t-3,\ t-12}$
CPI_SE	$\text{OMXS30}_{t-12,\ t-24}$	$\text{EDF}_{t-12,\ t-24}$
DEBT_SE	$\text{IPI\_SE}_{t-3,\ t-6}$	LRY_SE_{t-24}
EDF	OMXS30_{t-24}	
LRY_SE	$\text{DEBT\_SE}_{t-3,\ t-6,\ t-12}$	

C.2 European

Pairwise Granger Causality Tests
Date: 04/18/08 Time: 23:59
Sample: 1998:01 2007:12
Lags: 6

Null Hypothesis:	Obs	F-Statistic	Probability
IPI_EU does not Granger Cause HICP_EU	114	1.40307138195	0.220770021584
HICP_EU does not Granger Cause IPI_EU		1.74479254187	0.118260302191
IR_ECB does not Granger Cause HICP_EU	114	0.558450139484	0.762362523705
HICP_EU does not Granger Cause IR_ECB		0.487386442529	0.816418038310
IR_ECB does not Granger Cause IPI_EU	114	3.534336993	0.003232466273
IPI_EU does not Granger Cause IR_ECB		0.869843022	0.519889516702

Pairwise Granger Causality Tests
Date: 04/19/08 Time: 00:02
Sample: 1998:01 2007:12
Lags: 12

Null Hypothesis:	Obs	F-Statistic	Probability
IPI_EU does not Granger Cause HICP_EU	108	2.09902750794	0.025471574287
HICP_EU does not Granger Cause IPI_EU		1.80862507819	0.059822522972
IR_ECB does not Granger Cause HICP_EU	108	0.565097823506	0.86393888638
HICP_EU does not Granger Cause IR_ECB		0.487751107342	0.91657949977
IR_ECB does not Granger Cause IPI_EU	108	2.56116098445	0.00620213826
IPI_EU does not Granger Cause IR_ECB		1.97783648672	0.03653506838

Pairwise Granger Causality Tests
Date: 04/19/08 Time: 00:03
Sample: 1998:01 2007:12
Lags: 24

Null Hypothesis:	Obs	F-Statistic	Probability
IPI_EU does not Granger Cause HICP_EU	96	1.15642027313	0.3270389369
HICP_EU does not Granger Cause IPI_EU		0.937270123571	0.5565075833
IR_ECB does not Granger Cause HICP_EU	96	1.04404755986	0.4366969153
HICP_EU does not Granger Cause IR_ECB		0.737969050689	0.7871482575
IR_ECB does not Granger Cause IPI_EU	96	1.7753997914	0.0460160277
IPI_EU does not Granger Cause IR_ECB		0.809897673376	0.7068890291

C.3 Swedish and European

Dependent	X_1 (t-lag)	X_2 (t-lag)	X_3 (t-lag)	X_4 (t-lag)	X_5 (t-lag)
IPI_SE	$\text{ECB}_{t-3,\ t-6}$	$\text{OMXS30}_{t-6,\ t-12,\ t-24}$	$\text{IPI\_EU}_{t-3,\ t-12}$	$\text{EDF}_{t-3,\ t-12}$	USD_SEK_{t-12}
CPI_SE	$\text{EDF}_{t-12,\ t-24}$	$\text{OMXS30}_{t-12,\ t-24}$			
DEBT_SE	$\text{ECB}_{t-3,\ t-6,\ t-12,\ t-24}$	$\text{IPI\_SE}_{t-3,\ t-6}$	$\text{IPI\_EU}_{t-3,\ t-12}$	HICP_EU_{t-12}	LRY_SE_{t-24}
EDF	$\text{USD\_SEK}_{t-3,\ t-12}$	OMXS30_{t-24}			
LRY_SE	$\text{DEBT\_SE}_{t-3,\ t-6,\ t-12}$				
OMXS30	USD_SEK_{t-3}				
IPI_EU	$\text{ECB}_{t-3,\ t-6,\ t-12,\ t-24}$	$\text{OMXS30}_{t-3,\ t-6,\ t-12,\ t-24}$	EDF_{t-12}		
ECB	$\text{OMXS30}_{t-3,\ t-6,\ t-12,\ t-24}$	$\text{EDF}_{t-3,\ t-24}$	LRY_SE_{t-12}		
HICP_EU	IPI_EU_{t-12}				
USD_SEK	IPI_EU_{t-3}	DEBT_SE_{t-24}	HICP_EU_{t-24}		

Appendix D

D.1 Swedish

Vector Autoregression Estimates
Included observations: 107 after adjusting endpoints
Standard errors in ( ) & t-statistics in [ ]

	EDF(-1)	DEBT_SE(-1)	IPI_SE(-1)	CPI_SE(-1)	LRY_SE(-1)
EDF(-2)	0.894170	-1.638454	-9.443064	1.346332	0.128867
	(0.15944)	(1.67933)	(3.60161)	(0.45596)	(0.33525)
	[5.60816]	[-0.97566]	[-2.62190]	[2.95271]	[0.38440]
EDF(-3)	-0.016977	0.317630	-6.843319	-0.360943	-0.168930
	(0.20511)	(2.16030)	(4.63313)	(0.58655)	(0.43126)
	[-0.08277]	[0.14703]	[-1.47704]	[-0.61536]	[-0.39171]
…					
LRY_SE(-13)	0.125814	1.184173	-4.424959	0.325069	0.055623
	(0.06824)	(0.71872)	(1.54142)	(0.19514)	(0.14348)
	[1.84377]	[1.64761]	[-2.87071]	[1.66579]	[0.38768]
C	-0.289665	4.071744	-0.104324	-0.913098	-0.161713
	(0.16887)	(1.77869)	(3.81469)	(0.48294)	(0.35508)
	[-1.71528]	[2.28918]	[-0.02735]	[-1.89070]	[-0.45543]
R-squared	0.978222	0.975695	0.836691	0.817752	0.971985
Adj. R-squared	0.949816	0.943993	0.623678	0.580037	0.935444
Sum sq. resids	0.374044	41.49521	190.8609	3.059043	1.653675
S.E. equation	0.090174	0.949774	2.036946	0.257878	0.189603
F-statistic	34.43714	30.77711	3.927901	3.440053	26.59989
Log likelihood	150.7809	-101.1485	-182.7877	38.35147	71.25941
Akaike AIC	-1.678147	3.030813	4.556780	0.423337	-0.191765
Schwarz SC	-0.154385	4.554575	6.080542	1.947099	1.331997
Mean dependent	0.462613	4.807840	3.033727	0.135514	4.563271
S.D. dependent	0.402531	4.013281	3.320474	0.397932	0.746241
Determinant Residual Covariance		4.89E-05			
Log Likelihood (d.f. adjusted)		-228.0630			
Akaike Information Criteria		9.963795			
Schwarz Criteria		17.58261			

D.2 EDF with lags

Vector Autoregression Estimates
Included observations: 107 after adjusting endpoints
Standard errors in ( ) & t-statistics in [ ]

	EDF(-1)
EDF(-2)	1.039364
	(0.10276)
	[10.1147]
EDF(-3)	-0.015709
	(0.14835)
	[-0.10589]
…	
EDF(-13)	-0.083974
	(0.10242)
	[-0.81989]
C	0.015943
	(0.01297)
	[1.22917]
R-squared	0.963818
Adj. R-squared	0.959199
Sum sq. resids	0.621439
S.E. equation	0.081308
F-statistic	208.6646
Log likelihood	123.6208
Akaike AIC	-2.067679
Schwarz SC	-1.742943
Mean dependent	0.462613
S.D. dependent	0.402531

Appendix E

E.1 European

Vector Autoregression Estimates
Included observations: 107 after adjusting endpoints
Standard errors in ( ) & t-statistics in [ ]

	EDF(-1)	IPI_EU(-1)	IR_ECB(-1)	HICP_EU(-1)	USD_SEK(-1)
EDF(-2)	0.735438	-1.309311	-0.241545	0.305647	-0.032404
	(0.16318)	(1.86538)	(0.29058)	(0.34186)	(0.02140)
	[4.50683]	[-0.70190]	[-0.83125]	[0.89407]	[-1.51407]
EDF(-3)	-0.016495	-0.688537	-0.012718	0.196409	0.024720
	(0.18907)	(2.16128)	(0.33667)	(0.39609)	(0.02480)
	[-0.08724]	[-0.31858]	[-0.03778]	[0.49587]	[0.99690]
...					
USD_SEK(-13)	-0.860297	-10.78972	-1.475570	0.184110	0.125588
	(1.37323)	(15.6977)	(2.44530)	(2.87684)	(0.18011)
	[-0.62648]	[-0.68734]	[-0.60343]	[0.06400]	[0.69730]
C	-0.635145	0.399403	-1.149751	3.894824	-0.143645
	(0.55140)	(6.30312)	(0.98187)	(1.15514)	(0.07232)
	[-1.15189]	[0.06337]	[-1.17098]	[3.37173]	[-1.98629]
R-squared	0.983545	0.929717	0.989315	0.940778	0.987912
Adj. R-squared	0.962081	0.838044	0.975378	0.863532	0.972146
Sum sq. resids	0.282625	36.93138	0.896170	1.240383	0.004862
S.E. equation	0.078384	0.896022	0.139578	0.164210	0.010280
F-statistic	45.82425	10.14166	70.98578	12.17900	62.65874
Log likelihood	165.7743	-94.91487	104.0349	86.64496	383.1318
Akaike AIC	-1.958399	2.914297	-0.804390	-0.479345	-6.021156
Schwarz SC	-0.434636	4.438059	0.719372	1.044417	-4.497394
Mean dependent	0.462613	1.997196	4.004673	2.049875	0.914494
S.D. dependent	0.402531	2.226490	0.889523	0.444512	0.061598
Determinant Residual Covariance		1.69E-10			
Log Likelihood (d.f. adjusted)		444.5779			
Akaike Information Criteria		-2.608933			
Schwarz Criteria		5.009879			

E.2 European (EDF and USD_SE)

Vector Autoregression Estimates
Included observations: 107 after adjusting endpoints
Standard errors in ( ) & t-statistics in [ ]

	EDF(-1)	USD_SEK(-1)
EDF(-2)	0.813282	-0.015258
	(0.11202)	(0.01574)
	[7.25985]	[-0.96933]
...		
IR_ECB(-1)	-0.004413	0.001451
	(0.01564)	(0.00220)
	[-0.28223]	[0.66065]
R-squared	0.974888	0.978827
Adj. R-squared	0.966306	0.971591
Sum sq. resids	0.431303	0.008516
S.E. equation	0.073889	0.010382
F-statistic	113.5904	135.2674
Log likelihood	143.1605	353.1437
Akaike AIC	-2.152533	-6.077453
Schwarz SC	-1.453101	-5.378021
Mean dependent	0.462613	0.914494
S.D. dependent	0.402531	0.061598
Determinant Residual Covariance		5.87E-07
Log Likelihood (d.f. adjusted)		464.0183
Akaike Information Criteria		-7.626511
Schwarz Criteria		-6.227647

Appendix F

F.1 Swedish and European

Vector Autoregression Estimates
Date: 06/08/08 Time: 21:47
Sample(adjusted): 1999:02 2007:12
Included observations: 107 after adjusting endpoints
Standard errors in ( ) & t-statistics in [ ]

	EDF(-1)	DEBT_SE(-1)	IPI_SE(-1)	CPI_SE(-1)	LRY_SE(-1)	OMXS30(-1)	USD_SEK(-1)
EDF(-2)	0.301437	5.634154	-17.00126	0.661233	0.749731	-0.061355	0.033526
	(0.18120)	(2.60567)	(6.10688)	(0.68474)	(0.53610)	(0.05217)	(0.02941)
	[1.66355]	[2.16227]	[-2.78395]	[0.96567]	[1.39848]	[-1.17616]	[1.13987]
…							
HICP_EU(-1)	0.225218	1.068617	-2.920679	1.011623	0.133569	-0.033465	-0.010814
	(0.08418)	(1.21047)	(2.83696)	(0.31810)	(0.24905)	(0.02423)	(0.01366)
	[2.67551]	[0.88281]	[-1.02951]	[3.18023]	[0.53632]	[-1.38091]	[-0.79146]
R-squared	0.995579	0.990803	0.926202	0.935398	0.988740	0.996314	0.995026
Adj. R-squared	0.976568	0.951256	0.608868	0.657611	0.940321	0.980466	0.973636
Sum sq. resids	0.075934	15.70192	86.24886	1.084342	0.664677	0.006293	0.002001
S.E. equation	0.061617	0.886056	2.076642	0.232846	0.182302	0.017739	0.010002
F-statistic	52.36927	25.05366	2.918701	3.367321	20.42051	62.86551	46.51847
Log likelihood	236.0871	-49.15746	-140.2922	93.83785	120.0222	369.3215	430.6336
Akaike AIC	-2.786674	2.544999	4.248453	-0.127810	-0.617238	-5.277038	-6.423058
Schwarz SC	-0.613440	4.718234	6.421688	2.045424	1.555997	-3.103804	-4.249823
Mean dependent	0.462613	4.807840	3.033727	0.135514	4.563271	2.925145	0.914494
S.D. dependent	0.402531	4.013281	3.320474	0.397932	0.746241	0.126921	0.061598
Determinant Residual Covariance	1.63E-13						
Log Likelihood (d.f. adjusted)	512.3691						
Akaike Information Criteria	1.806186						
Schwarz Criteria	17.01883						

F.2 New endogenous and new exogenous

Vector Autoregression Estimates
Date: 06/08/08 Time: 21:53
Sample(adjusted): 1999:02 2007:12
Included observations: 107 after adjusting endpoints
Standard errors in ( ) & t-statistics in [ ]

	EDF(-1)	LRY_SE(-1)	OMXS30(-1)	USD_SEK(-1)
EDF(-2)	0.782895	0.356902	-0.095294	0.002927
	(0.15490)	(0.36913)	(0.04114)	(0.02106)
	[5.05415]	[0.96687]	[-2.31639]	[0.13897]
EDF(-3)	0.070582	-0.369803	0.060340	0.029465
	(0.19380)	(0.46183)	(0.05147)	(0.02636)
	[0.36420]	[-0.80073]	[1.17233]	[1.11800]
…				
CPI_SE(-1)	0.025719	0.077199	-0.000604	-8.33E-05
	(0.02634)	(0.06277)	(0.00700)	(0.00358)
	[0.97640]	[1.22987]	[-0.08628]	[-0.02324]
R-squared	0.981266	0.969046	0.986709	0.985205
Adj. R-squared	0.961811	0.936901	0.972907	0.969841
Sum sq. resids	0.321764	1.827195	0.022695	0.005950
S.E. equation	0.078662	0.187452	0.020891	0.010697
F-statistic	50.43874	30.14622	71.48866	64.12438
Log likelihood	158.8355	65.92110	300.7000	372.3196
Akaike AIC	-1.940851	-0.204133	-4.592523	-5.931208
Schwarz SC	-0.566967	1.169751	-3.218639	-4.557324
Mean dependent	0.462613	4.563271	2.925145	0.914494
S.D. dependent	0.402531	0.746241	0.126921	0.061598
Determinant Residual Covariance		6.60E-12		
Log Likelihood (d.f. adjusted)		769.9561		
Akaike Information Criteria		-10.27955		
Schwarz Criteria		-4.784017		

F.3 The representation of the VAR(12)

EDF(-1) =
0.7781*EDF(-2) + 0.0350*EDF(-3) + 0.3785*EDF(-4) - 0.4102*EDF(-5) + 0.1458*EDF(-6) - 0.3319*EDF(-7) - 0.0664*EDF(-8) - 0.0172*EDF(-9) + 0.2445*EDF(-10) - 0.1555*EDF(-11) - 0.1729*EDF(-12) + 0.1463*EDF(-13) -

0.0062*LRY_SE(-2) - 0.0377*LRY_SE(-3) + 0.0838*LRY_SE(-4) - 0.0158*LRY_SE(-5) - 0.0290*LRY_SE(-6) - 0.0304*LRY_SE(-7) + 0.0093*LRY_SE(-8) - 0.0425*LRY_SE(-9) + 0.0132*LRY_SE(-10) + 0.0189*LRY_SE(-11) - 0.0907*LRY_SE(-12) + 0.1376*LRY_SE(-13) +

0.1681*OMXS30(-2) - 0.1008*OMXS30(-3) - 0.1223*OMXS30(-4) + 0.2272*OMXS30(-5) - 0.4203*OMXS30(-6) + 0.0692*OMXS30(-7) - 0.4546*OMXS30(-8) + 0.2616*OMXS30(-9) + 0.5789*OMXS30(-10) - 1.0297*OMXS30(-11) - 0.5759*OMXS30(-12) + 0.7619*OMXS30(-13) –

0.7773*USD_SEK(-2) - 0.0950*USD_SEK(-3) - 0.0583*USD_SEK(-4) + 0.5832*USD_SEK(-5) - 0.0467*USD_SEK(-6) - 2.3026*USD_SEK(-7) + 2.7647*USD_SEK(-8) - 0.6494*USD_SEK(-9) - 0.7830*USD_SEK(-10) + 2.7372*USD_SEK(-11) + 1.1226*USD_SEK(-12) - 0.7592*USD_SEK(-13) –

0.0027*DEBT_SE(-1) + 0.0051*IPI_SE(-1) + 0.0327*CPI_SE(-1) - 0.0049*IPI_EU(-1) + 0.0436*IR_ECB(-1) + 0.1081*HICP_EU(-1)

LRY_SE(-1) = 0.3796*EDF(-2) - 0.1995*EDF(-3) + …..+ 0.0717*IR_ECB(-1) - 0.1584*HICP_EU(-1)

OMXS30(-1) = - 0.0929*EDF(-2) + 0.0774*EDF(-3) + … - 0.010*IR_ECB(-1) - 0.020*HICP_EU(-1)

USD_SEK(-1) = 0.0022*EDF(-2) + 0.0246*EDF(-3) - … + 0.0014*IR_ECB(-1) - 0.005*HICP_EU(-1)

Appendix G

G.1 Cointegration

Vector Error Correction Estimates
Included observations: 107 after adjusting endpoints
Standard errors in ( ) & t-statistics in [ ]

Cointegrating Eq:	CointEq1			
EDF(-1)	1.000000			
USD_SEK(-1)	-6.165181			
	(1.87561)			
	[-3.28703]			
OMXS30(-1)	-0.749733			
	(0.66349)			
	[-1.12999]			
LRY_SE(-1)	-0.053974			
	(0.10149)			
	[-0.53180]			
C	7.614784			
Error Correction:	**D(EDF)**	**D(USD_SEK)**	**D(OMXS30)**	**D(LRY_SE)**
CointEq1	-0.125683	-0.014436	0.030959	-0.282937
	(0.06190)	(0.00826)	(0.01721)	(0.14834)
	[-2.03042]	[-1.74881]	[1.79901]	[-1.90736]
D(EDF(-1))	0.016511	0.011549	-0.096626	0.387232
	(0.14435)	(0.01925)	(0.04013)	(0.34593)
	[0.11438]	[0.59990]	[-2.40777]	[1.11939]
D(EDF(-2))	0.016510	0.048485	-0.022471	0.320531
	(0.15155)	(0.02021)	(0.04213)	(0.36318)
	[0.10894]	[2.39899]	[-0.53335]	[0.88258]
. . .				
D(LRY_SE(-12))	0.104537	9.25E-05	0.019128	-0.344432
	(0.05779)	(0.00771)	(0.01607)	(0.13849)
	[1.80887]	[0.01200]	[1.19056]	[-2.48698]
C	-0.001049	-0.002096	0.002552	-0.025619
	(0.01049)	(0.00140)	(0.00292)	(0.02513)
	[-0.10001]	[-1.49852]	[0.87551]	[-1.01945]
R-squared	0.554780	0.486629	0.521521	0.505512
Adj. R-squared	0.172047	0.045310	0.110197	0.080426
Sum sq. resids	0.356017	0.006332	0.027516	2.044582
S.E. equation	0.079031	0.010540	0.021971	0.189393
F-statistic	1.449523	1.102670	1.267909	1.189200
Log likelihood	153.4235	368.9964	290.3955	59.90709
Akaike AIC	-1.933150	-5.962549	-4.493374	-0.185179
Schwarz SC	-0.684164	-4.713563	-3.244388	1.063806
Mean dependent	-0.000865	-0.000765	0.001738	0.003131
S.D. dependent	0.086855	0.010787	0.023292	0.197502
Determinant Residual Covariance		7.35E-12		
Log Likelihood		898.9782		
Log Likelihood (d.f. adjusted)		764.2058		
Akaike Information Criteria		-10.47114		
Schwarz Criteria		-5.375276		

G.2 Cointegration without LRY_SE

Vector Error Correction Estimates
Included observations: 107 after adjusting endpoints
Standard errors in ( ) & t-statistics in [ ]

Cointegrating Eq:	CointEq1		
EDF(-1)	1.000000		
USD_SEK(-1)	-4.331185 (0.79455) [-5.45114]		
OMXS30(-1)	0.835978 (0.40039) [2.08793]		
C	1.052875		
Error Correction:	**D(EDF)**	**D(USD_SEK)**	**D(OMXS30)**
CointEq1	-0.202110 (0.08172) [-2.47323]	-0.016256 (0.01141) [-1.42430]	-0.004237 (0.02320) [-0.18263]
D(EDF(-1))	0.015652 (0.13404) [0.11677]	0.000895 (0.01872) [0.04782]	-0.087389 (0.03805) [-2.29655]
. . .			
C	-0.001997 (0.00843) [-0.23681]	-0.001313 (0.00118) [-1.11515]	0.000686 (0.00239) [0.28673]
R-squared	0.481352	0.344091	0.418783
Adj. R-squared	0.203236	-0.007628	0.107116
Sum sq. resids	0.414733	0.008090	0.033424
S.E. equation	0.077528	0.010828	0.022009
F-statistic	1.730760	0.978312	1.343689
Log likelihood	145.2563	355.8876	279.9891
Akaike AIC	-2.004791	-5.941824	-4.523161
Schwarz SC	-1.055562	-4.992596	-3.573932
Mean dependent	-0.000865	-0.000765	0.001738
S.D. dependent	0.086855	0.010787	0.023292
Determinant Residual Covariance		2.36E-10	
Log Likelihood		800.9752	
Log Likelihood (d.f. adjusted)		730.5603	
Akaike Information Criteria		-11.46842	
Schwarz Criteria		-8.545791	

G.3 Lag Length Criteria

VAR Lag Order Selection Criteria
Endogenous variables: EDF(-1) OMXS30(-1) USD_SEK(-1)
Exogenous variables: C
Included observations: 107

Lag	LogL	LR	FPE	AIC	SC	HQ
0	243.1747	NA	2.25E-06	-4.489248	-4.414308	-4.458868
1	728.9178	935.1689	3.04E-10	-13.40033	-13.10058*	-13.27882*
2	741.9785	24.41247	2.82E-10*	-13.47623*	-12.95166	-13.26358
3	746.5357	8.262582	3.07E-10	-13.39319	-12.64380	-13.08940
4	759.4786	22.74077*	2.85E-10	-13.46689	-12.49268	-13.07196
5	764.3756	8.329580	3.09E-10	-13.39020	-12.19117	-12.90413
6	769.1020	7.774319	3.36E-10	-13.31032	-11.88648	-12.73311
7	776.6999	12.07142	3.48E-10	-13.28411	-11.63545	-12.61577
8	779.5427	4.357136	3.93E-10	-13.16902	-11.29554	-12.40954
9	785.0997	8.205602	4.24E-10	-13.10467	-11.00637	-12.25405
10	793.1138	11.38463	4.38E-10	-13.08624	-10.76313	-12.14448
11	794.6932	2.155012	5.12E-10	-12.94754	-10.39961	-11.91464
12	800.6573	7.803461	5.53E-10	-12.89079	-10.11804	-11.76676

* indicates lag order selected by the criterion
LR: sequential modified LR test statistic (each test at 5% level)
FPE: Final prediction error
AIC: Akaike information criterion
SC: Schwarz information criterion
HQ: Hannan-Quinn information criterion

Appendix H

H.1 VAR(4)

Vector Autoregression Estimates
Included observations: 115 after adjusting endpoints
Standard errors in ( ) & t-statistics in [ ]

	EDF(-1)	OMXS30(-1)	USD_SEK(-1)
EDF(-2)	0.821908	-0.037050	-0.012182
	(0.10778)	(0.03155)	(0.01439)
	[7.62586]	[-1.17437]	[-0.84641]
…			
USD_SEK(-5)	0.417614	-0.037019	0.040031
	(0.79555)	(0.23287)	(0.10624)
	[0.52494]	[-0.15897]	[0.37681]
C	-0.419699	0.200435	-0.018049
	(0.26956)	(0.07891)	(0.03600)
	[-1.55697]	[2.54019]	[-0.50141]
R-squared	0.966159	0.970756	0.973276
Adj. R-squared	0.962178	0.967316	0.970132
Sum sq. resids	0.605705	0.051899	0.010801
S.E. equation	0.077060	0.022557	0.010290
F-statistic	242.6742	282.1616	309.5692
Log likelihood	138.4840	279.7670	370.0216
Akaike AIC	-2.182330	-4.639426	-6.209071
Schwarz SC	-1.872034	-4.329130	-5.898774
Mean dependent	0.441269	2.919396	0.913472
S.D. dependent	0.396238	0.124770	0.059543

Determinant Residual Covariance	2.26E-10
Log Likelihood (d.f. adjusted)	787.5108
Akaike Information Criteria	-13.01758
Schwarz Criteria	-12.08669

H.2 Cointegration

Vector Error Correction Estimates
Included observations: 115 after adjusting endpoints
Standard errors in ( ) & t-statistics in [ ]

Cointegrating Eq:	CointEq1
EDF(-1)	1.000000
OMXS30(-1)	0.529676
	(0.54530)
	[0.97134]
USD_SEK(-1)	-5.055039
	(1.18115)
	[-4.27976]
C	2.630035

Error Correction:	D(EDF)	D(OMXS30)	D(USD_SEK)
CointEq1	-0.097167	0.008094	-0.009322
	(0.03220)	(0.00975)	(0.00436)
	[-3.01720]	[0.83054]	[-2.13761]
…			
R-squared	0.272863	0.218242	0.170139
Adj. R-squared	0.179272	0.117620	0.063325
Sum sq. resids	0.586089	0.053671	0.010746
S.E. equation	0.076177	0.023052	0.010315
F-statistic	2.915461	2.168925	1.592858
Log likelihood	140.3769	277.8363	370.3138
Akaike AIC	-2.197860	-4.588457	-6.196762
Schwarz SC	-1.863694	-4.254291	-5.862597
Mean dependent	0.000279	0.001352	-0.000647
S.D. dependent	0.084086	0.024540	0.010658

Determinant Residual Covariance	2.36E-10
Log Likelihood	807.4200
Log Likelihood (d.f. adjusted)	785.0275
Akaike Information Criteria	-12.87004
Schwarz Criteria	-11.79594

Appendix I

I.1 Autocorrelation

VAR Residual Serial Correlation LM Tests
H0: no serial correlation at lag order h
Included observations: 115

Lags	LM-Stat	Prob
1	8.091995	0.5249
2	8.120518	0.5220
3	2.770937	0.9727
4	6.596640	0.6790
5	5.238981	0.8130
6	16.87637	0.0507
7	5.008979	0.8335
8	3.744035	0.9274
9	11.16597	0.2645
10	4.521092	0.8739
11	10.83959	0.2869
12	9.037544	0.4338

Probs from chi-square with 9 df.

I.2 Heteroskedasticity

VAR Residual Heteroskedasticity Tests: No Cross Terms (only levels and squares)
Included observations: 115

Joint test:

Chi-sq	df	Prob.
200.7203	144	0.0013

Individual components:

Dependent	R-squared	F(24,90)	Prob.	Chi-sq(24)	Prob.
res1*res1	0.638597	6.626236	0.0000	73.43868	0.0000
res2*res2	0.199468	0.934385	0.5571	22.93883	0.5234
res3*res3	0.264003	1.345130	0.1596	30.36036	0.1731
res2*res1	0.486241	3.549148	0.0000	55.91776	0.0002
res3*res1	0.320524	1.768959	0.0286	36.86027	0.0452
res3*res2	0.256043	1.290616	0.1941	29.44497	0.2038

I.3 Normal distributed

VAR Residual Normality Tests
Orthogonalization: Cholesky (Lutkepohl)
H0: residuals are multivariate normal
Included observations: 115

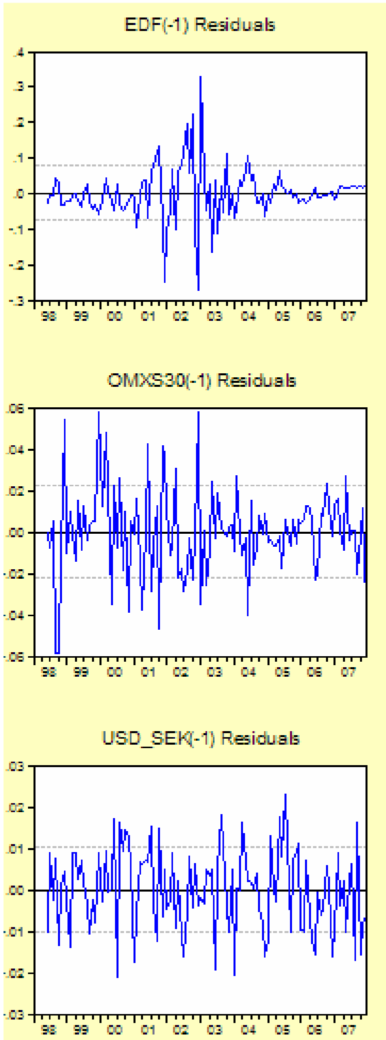
Component	Skewness	Chi-sq	df	Prob.
1	0.322466	1.993034	1	0.1580
2	-0.039619	0.030085	1	0.8623
3	-0.077872	0.116227	1	0.7332
Joint		2.139346	3	0.5440

Component	Kurtosis	Chi-sq	df	Prob.
1	6.917928	73.55287	1	0.0000
2	3.523033	1.310826	1	0.2522
3	1.872926	6.086831	1	0.0136
Joint		80.95052	3	0.0000

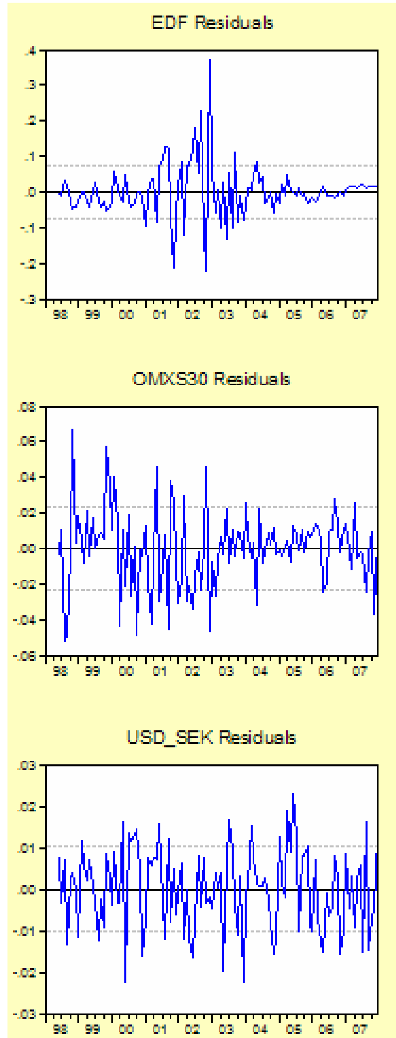
Component	Jarque-Bera	df	Prob.
1	75.54590	2	0.0000
2	1.340910	2	0.5115
3	6.203057	2	0.0450
Joint	83.08987	6	0.0000

Appendix J

J.1 VAR(4)



J.2 VEC(4)



Appendix K

Included observations: 115 after adjusting endpoints
Trend assumption: Linear deterministic trend
Series: EDF OMXS30 USD_SEK
Lags interval (in first differences): 1 to 4

Unrestricted Cointegration Rank Test

Hypothesized No. of CE(s)	Eigenvalue	Trace Statistic	5 Percent Critical Value	1 Percent Critical Value
None	0.129797	25.25331	29.68	35.65
At most 1	0.073068	9.265055	15.41	20.04
At most 2	0.004680	0.539437	3.76	6.65

*(**) denotes rejection of the hypothesis at the 5%(1%) level
Trace test indicates no cointegration at both 5% and 1% levels

Hypothesized No. of CE(s)	Eigenvalue	Max-Eigen Statistic	5 Percent Critical Value	1 Percent Critical Value
None	0.129797	15.98825	20.97	25.52
At most 1	0.073068	8.725619	14.07	18.63
At most 2	0.004680	0.539437	3.76	6.65

*(**) denotes rejection of the hypothesis at the 5%(1%) level
Max-eigenvalue test indicates no cointegration at both 5% and 1% levels

Unrestricted Cointegrating Coefficients (normalized by b'*S11*b=I):

EDF	OMXS30	USD_SEK
-4.533587	-2.401330	22.91746
-2.862803	-9.885137	-5.674693
-1.329320	-8.488865	16.32170

Unrestricted Adjustment Coefficients (alpha):

D(EDF)	0.021433	-0.007736	0.002367
D(OMXS30)	-0.001785	0.005693	8.95E-05
D(USD_SEK)	0.002056	0.000518	-0.000536

1 Cointegrating Equation(s): Log likelihood 807.4200

Normalized cointegrating coefficients (std.err. in parentheses)

EDF	OMXS30	USD_SEK
1.000000	0.529676	-5.055039
	(0.54530)	(1.18115)

Adjustment coefficients (std.err. in parentheses)

D(EDF)	-0.097167
	(0.03220)
D(OMXS30)	0.008094
	(0.00975)
D(USD_SEK)	-0.009322
	(0.00436)

2 Cointegrating Equation(s): Log likelihood 811.7828

Normalized cointegrating coefficients (std.err. in parentheses)

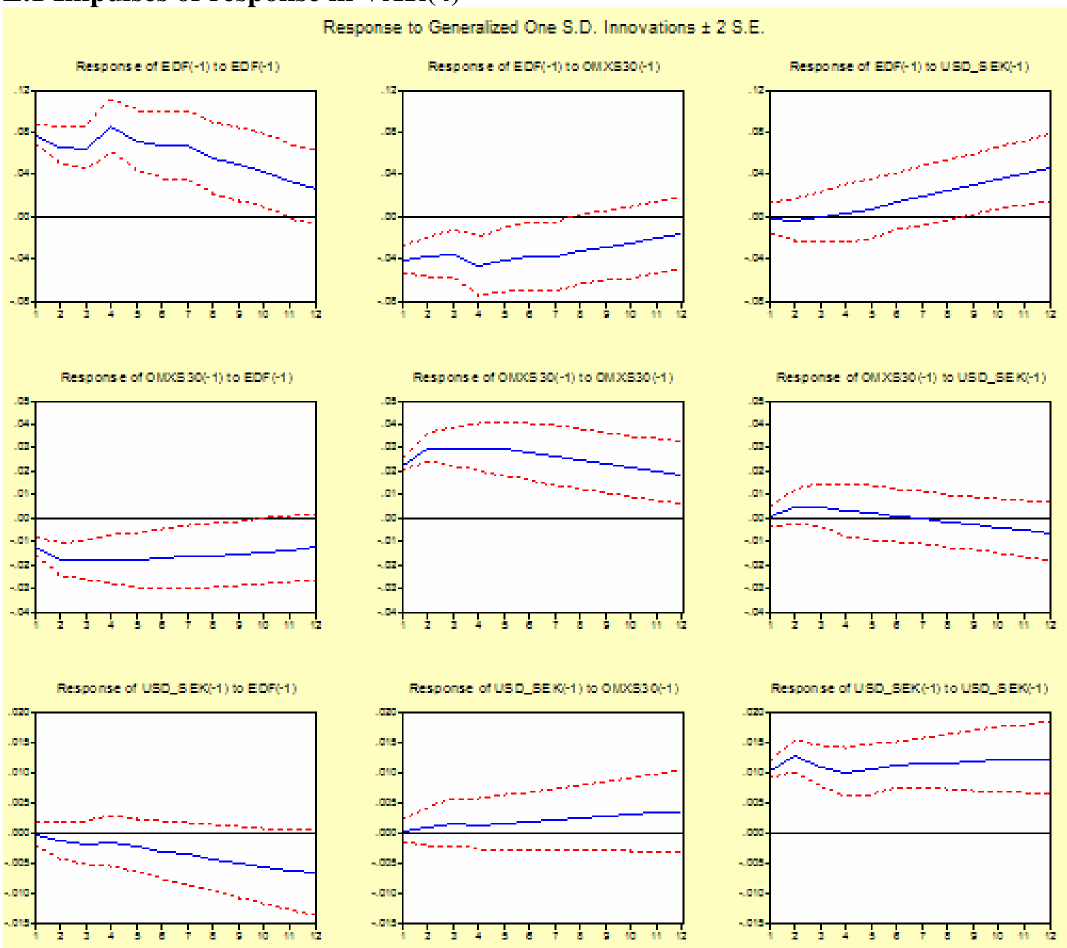
EDF	OMXS30	USD_SEK
1.000000	0.000000	-6.330134
		(1.43581)
0.000000	1.000000	2.407313
		(0.94531)

Adjustment coefficients (std.err. in parentheses)

D(EDF)	-0.075020	0.025007
	(0.03786)	(0.07184)
D(OMXS30)	-0.008205	-0.051993
	(0.01112)	(0.02109)
D(USD_SEK)	-0.010805	-0.010059
	(0.00515)	(0.00977)

Appendix L

L.1 Impulses of response in VAR(4)



L.2 VAR(4) with exogenous variables

EDF(-1) = 0.8286821669*EDF(-2) + 0.1356831195*EDF(-3) + 0.2996959462*EDF(-4) - 0.3651814881*EDF(-5) - 0.2165066989*OMXS30(-2) + 0.3565123813*OMXS30(-3) - 0.2231763878*OMXS30(-4) - 0.01685238133*OMXS30(-5) - 0.2923743802*USD_SEK(-2) + 0.6725661359*USD_SEK(-3) - 0.2543224459*USD_SEK(-4) + 0.174522778*USD_SEK(-5) - 0.002670799716*DEBT_SE(-1) - 0.003461571454*IPI_EU(-1) + 0.02010515239*IR_ECB(-1)

OMXS30(-1) = - 0.0378743053*EDF(-2) + 0.04889037175*EDF(-3) - 0.007805183804*EDF(-4) - 0.004940447056*EDF(-5) + 1.251805879*OMXS30(-2) - 0.3017611636*OMXS30(-3) + 0.07082979211*OMXS30(-4) - 0.008550513485*OMXS30(-5) + 0.4352067642*USD_SEK(-2) - 0.6043190211*USD_SEK(-3) + 0.1303681512*USD_SEK(-4) + 0.03752679901*USD_SEK(-5) - 7.558815139e-05*DEBT_SE(-1) + 0.0006251397281*IPI_EU(-1) - 0.008347344875*IR_ECB(-1)

USD_SEK(-1) = - 0.01126792631*EDF(-2) + 0.009254499257*EDF(-3) + 0.003766781357*EDF(-4) - 0.01365423503*EDF(-5) + 0.01167103384*OMXS30(-2) + 0.001989186535*OMXS30(-3) - 0.02720624863*OMXS30(-4) + 0.01073056658*OMXS30(-5) + 1.202958783*USD_SEK(-2) - 0.4451792002*USD_SEK(-3) + 0.235388234*USD_SEK(-4) + 0.01854593318*USD_SEK(-5) - 0.0004721184365*DEBT_SE(-1) - 0.0004363534817*IPI_EU(-1) + 0.001246708004*IR_ECB(-1)

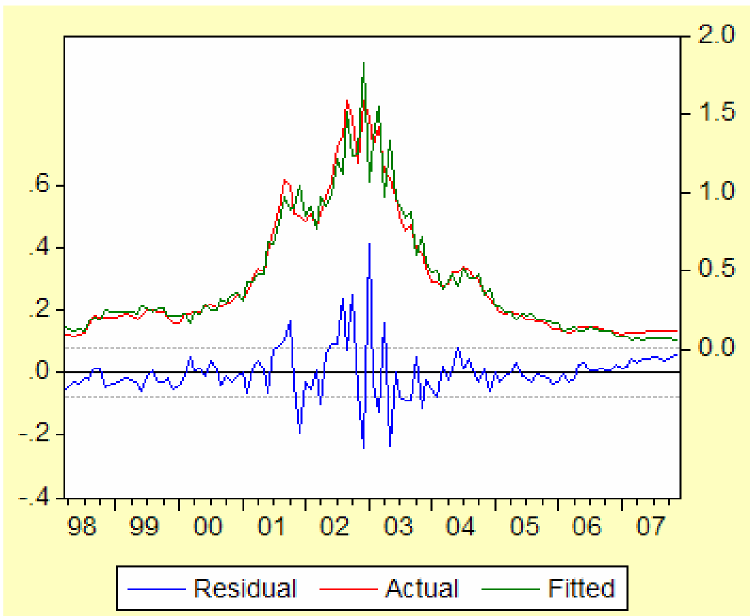
L.3 Equation for scenario analysis

Dependent Variable: EDF(-1)
Method: Least Squares
Sample(adjusted): 1998:03 2007:12
Included observations: 118 after adjusting endpoints

Variable	Coefficient	Std. Error	t-Statistic	Prob.
EDF(-2)	0.900297	0.026957	33.39750	0.0000
OMXS30(-1)	-0.164380	0.043605	-3.769771	0.0003
USD_SEK(-1)	0.572595	0.148495	3.855996	0.0002

R-squared	0.961337	Mean dependent var	0.432294
Adjusted R-squared	0.960665	S.D. dependent var	0.395086
S.E. of regression	0.078358	Akaike info criterion	-2.229965
Sum squared resid	0.706095	Schwarz criterion	-2.159524
Log likelihood	134.5680	Durbin-Watson stat	2.038368

L.4



L.5

Date: 06/10/08 Time: 23:30
Sample: 1998:03 2007:12
Included observations: 118

<table>
<tr><th>Autocorrelation</th><th>Partial Correlation</th><th></th><th>AC</th><th>PAC</th><th>Q-Stat</th><th>Prob</th></tr>
<tr><td>.|. |</td><td>.|. |</td><td>1</td><td>-0.024</td><td>-0.024</td><td>0.0686</td><td>0.793</td></tr>
<tr><td>.|. |</td><td>.|. |</td><td>2</td><td>0.057</td><td>0.057</td><td>0.4679</td><td>0.791</td></tr>
<tr><td>.|*** |</td><td>.|*** |</td><td>3</td><td>0.344</td><td>0.348</td><td>15.008</td><td>0.002</td></tr>
<tr><td>**|. |</td><td>**|. |</td><td>4</td><td>-0.203</td><td>-0.210</td><td>20.142</td><td>0.000</td></tr>
<tr><td>.|* |</td><td>.|* |</td><td>5</td><td>0.125</td><td>0.096</td><td>22.111</td><td>0.000</td></tr>
<tr><td>.|. |</td><td>*|. |</td><td>6</td><td>0.011</td><td>-0.103</td><td>22.127</td><td>0.001</td></tr>
<tr><td>*|. |</td><td>.|. |</td><td>7</td><td>-0.167</td><td>-0.041</td><td>25.705</td><td>0.001</td></tr>
<tr><td>.|. |</td><td>*|. |</td><td>8</td><td>0.035</td><td>-0.086</td><td>25.861</td><td>0.001</td></tr>
<tr><td>*|. |</td><td>.|. |</td><td>9</td><td>-0.084</td><td>0.000</td><td>26.779</td><td>0.002</td></tr>
<tr><td>*|. |</td><td>.|. |</td><td>10</td><td>-0.074</td><td>-0.031</td><td>27.489</td><td>0.002</td></tr>
<tr><td>.|. |</td><td>.|. |</td><td>11</td><td>0.030</td><td>0.027</td><td>27.604</td><td>0.004</td></tr>
<tr><td>.|* |</td><td>.|* |</td><td>12</td><td>0.069</td><td>0.144</td><td>28.248</td><td>0.005</td></tr>
</table>