VILNIUS GEDIMINAS TECHNICAL UNIVERSITY
FACULTY OF ENVIRONMENTAL ENGINEERING
DEPARTMENT OF ROADS

Povilas Goberis

THE USE OF TENSEGRITY SYSTEMS IN SEARCH FOR NEW FORMS OF CABLE-STAYED BRIDGES
INTEGRUOTO TEMPIMO SISTEMŲ IEŠKANT NAUJOS VANTINIŲ TILTŲ FORMOS TYRIMAS

Master‘s degree Thesis

Study programme of Innovative Road and Bridge Engineering, state code 628H20001

Study field of Civil Engineering

Vilnius, 2017

VILNIUS GEDIMINAS TECHNICAL UNIVERSITY
FACULTY OF ENVIRONMENTAL ENGINEERING
DEPARTMENT OF ROADS

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2017 01 04
(Date)

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Supervisor Dr. Ieva Misiūnaitė (Title, Name, Surname) ______ (Signature) 2017 01 04 (Date)

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(the document of Declaration of Authorship in the Final Degree Project)

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(Study programme, academic group no.)

DECLARATION OF AUTHORSHIP
IN THE FINAL DEGREE PROJECT

January 4, 2017

I declare that my Final Degree Project entitled „The Use of Tensegrity Systems in Search for New Forms of Cable-stayed Bridges“ is entirely my own work. The title was confirmed on November 14, 2016 by Faculty Dean's order No. 219ap. I have clearly signalled the presence of quoted or paraphrased material and referenced all sources.

I have acknowledged appropriately any assistance I have received by the following professionals/advisers: Dr Ieva Misiūnaitė.

The academic supervisor of my Final Degree Project is Dr Ieva Misiūnaitė.

No contribution of any other person was obtained, nor did I buy my Final Degree Project.

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VILNIUS GEDIMINAS TECHNICAL UNIVERSITY
FACULTY OF ENVIRONMENTAL ENGINEERING
DEPARTMENT OF ROADS

Construction engineering study field

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(Name, Surname)

2016 01 04

(Date)

OBJECTIVES FOR MASTER THESIS

..................No. ....

Vilnius

For student ....Povilas Goberis..
(Name, Surname)

Master Thesis title: The use of tensegrity systems in search for new forms of cable-stayed bridges...

Approved on 14, November 2016 by Dean's decree No. 219 ap

The Final work has to be completed by 11 January, 2017

THE OBJECTIVES:

To review evolution and concept of cable-stayed bridges and tensegrity systems, as well as, relevant configurations of tensegrity systems applicable in those days infrastructure. Examine the possibility of using tensegrity systems as a tool in order to solve the form-finding problems of cable-stayed bridges. Review the form-finding methods applicable for the tensegrity systems and choose the most appropriate one in order to find a new form of cable-stayed bridge with arbitrary structural arrangement. To write the algorithm for the considering form-finding problem by using arbitrary numerical computing environment. The suggested computational technique should be verified by finite element analysis using one of the commercial softwares. Thesis has to be finalised with the discussion of the results obtained by comparison analysis and concluded by main findings and suggestions for the future research and developments.

....

Consultants of the Master Thesis: ....

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(Title, Name, Surname)

Academic Supervisor .... dr. Ieva Misiūnaitė.....................
(Signature) (Title, Name, Surname)

Objectives accepted as a quidance for my Master Thesis

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Master Degree Studies **Innovative Road and Bridge Engineering** study programme Master Graduation Thesis 3 (Roads and Bridges)

Title	**The Use of Tensegrity Systems in Search for New Forms of Cable-stayed Bridges**
Author	**Povilas Goberis**
Academic supervisor	**Ieva Misiūnaitė**

Thesis language: English

Annotation

Cable-stayed bridges are one of the most expressive and technically effective bridges. Cable-stayed bridges distinguished by a variety of structural forms, which allows the design of technically and economically rational and architecturally expressive structures. The use of tensegrity systems in cable-stayed bridge design can improve these properties. However, Despite the advantages of tensegrity systems their use in large structures is limited due to excessive deformations under heavy loading and the current lack of analytical tools to study and design tensegrity structures. Development of new methods, experimental and analytical research are subjects of extensive studies all around the world. This thesis is an analysis of application of tensegrity systems for the form-finding of cable-stayed bridges. In the first section of the thesis brief historical review and concept of both, cable-stayed bridges and tensegrity systems are presented. Further review of tensegrity systems characteristics, applications, and existing examples are performed. The second section of the thesis is the analysis of form-finding methods of tensegrity systems. An approach of form-finding is suggested and computational algorithm has been written. In the third section form-finding of cable-stayed bridge composed of tensegrity modules was performed. Obtained geometry of the bridge further analysed with finite element program. The last section of the thesis concludes the results of the study. Furthermore, it provides the findings and recommendations for future research.

The main parts of the thesis are: introduction, review of cable-stayed bridges and tensegrity systems, analysis of the form-finding methods of tensegrity systems, analysis of cable-stayed bridge by using tensegrity system, conclusions, references and appendices.

Thesis consists of 82 pages of text, 80 figures, 14 tables, 114 references. Appendices.

Keywords: cable-stayed bridge, tensegrity system, form-finding, finite element analysis

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Antrosios pakopos studijų **Inovatyvios kelių ir tiltų inžinerijos** programos magistro baigiamasis darbas 3 (keliai ir tiltai)

Pavadinimas	**Integruoto tempimo sistemų ieškant naujos vantinių tiltų formos tyrimas**
Autorius	**Povilas Goberis**
Vadovas	**Ieva Misiūnaitė**

Kalba: anglų

Anotacija

Vieni iš išraiškingiausių ir techniniu požiūriu veiksmingiausių tiltų yra vantiniai. Vantiniai tiltai pasižymi konstrukcinių formų įvairove, kuri leidžia projektuoti techniškai ir ekonomiškai racionalius bei architektūriškai išraiškingus statinius. Integruoto tempimo sistemų panaudojimas vantinių tiltų konstrukcijose gali pagerinti šias savybes. Tačiau nepaisant šių sistemų pranašumų dėl pernelyg didelių deformacijų esant didelėms apkrovoms ir dabartinės analitinių tyrimų ir modeliavimo įrankių stokos jų panaudojimas didelėms konstrukcijoms yra ribotas. Naujų metodų kūrimas, bei eksperimentiniai ir analitiniai tyrimai yra išsamių tyrimų visame pasaulyje subjektai. Baigiamajame darbe analizuojamas integruoto tempimo sitemų panaudojimas vantinių tiltų formos paieškoje. Pirmajame skyriuje pateikta trumpa vantinių tiltų bei integruoto tempimo sistemų raida bei sąvokos. Atlikta tolimesnė integruoto tempimo sistemų pagrindinių savybių, panaudojimo galimybių ir egzistuojančių šių sistemų panaudojimo pavyzdžių analizė. Antrajame skyriuje atlikta integruotų sistemų formos paieškos metodų analizė. Pasiūlytas skaičiavimo metodas ir pateiktas skaičiavimo algoritmas. Trečiajame skyriuje atlikta vantinio tilto, sudaryto iš integruoto tempimo modulių, formos paieška. Gauta tilto geometrija toliau išanalizuota su baigtinių elementų programa. Išnagrinėjus gautus rezultatus, paskutiniame skyriuje pateikiamos baigiamojo darbo išvados, bei rekomendacijos tolimesniems tyrimams.

Darbą sudaro įvadas, vantinių tiltų ir integruoto tempimo sistemų apžvalga, integruoto tempimo sistemų formos paieškos metodų analizė, vantinio tilto analizė panaudojant integruoto tempimo sistemą, išvados, literatūros sąrašas ir priedai.

Darbo apimtis – 82 puslapiai teksto be priedų, 80 paveikslų, 14 lentelių, 114 literatūros šaltinių. Atskirai pridedami darbo priedai.

Prasminiai žodžiai: vantinis tiltas, integruoto tempimo sistema, formos paieška, baigtinių elementų analizė

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ABBREVIATIONS

AM – Algebraic Method
AS – Analytical Solution
DE – Differential Equations
DR – Dynamic Relaxation
EFDM – Extended Force Density Method
EM – Energy Method
FDM – Force Density Method
FEA – Finite Element Analysis
FEM – Finite Element Modelling
GA – Genetic Algorithm
NLP – Non-Linear Programming
NM – Numerical Method
RCM – Reduced Coordinates Method
SA – Successive Approximation Method
SLS – Serviceability Limit State
SQP – Sequential Quadratic Programming
ULS – Ultimate Limit State

INTRODUCTION

The Investigated Problem

Nowadays bridges are important transportation system structures, without these structures traffic of vehicles, trains, and pedestrians is impossible. Modern bridge design is characterized by the use of new materials and construction techniques, expressive architectural forms and unique design solutions.

Cable-stayed bridges stand as a bridge type distinguished by its expressive forms and technical characteristics. Main components of this kind of bridges are prestressed cable stays made of high strength steel, which are connected to the pylons and hold deck structure. Cable-stayed bridges characterized by a variety of structural arrangements allowing the design of technically and economically rational and architecturally expressive structures. Nevertheless, all these features can be improved by applying tensegrity systems.

The origins of tensegrity are arguable, Richard Buckminster Fuller (1962), David Georges Emmerich (1964), and Kenneth D. Snelson (1965) all have been considered as inventors of tensegrity. Each of them granted their patents in almost the same time. Tensegrity systems are technical structures consisting of tension and compression elements which are lightweight, deployable, energy efficient, and highly controllable. Tensegrity structure is an architype of incessant tension and discontinuous compression. These structures mostly embed integrated struts and cables, which are all loaded axially and do not experience any bending moments.

In comparison to the conventional structures tensegrity systems surprise and fascinate with exceptional structural forms. The initial geometry of this kind of structures can be defined trough so called form-finding process. Form-finding means a procedure for seeking, evaluating and obtaining geometry.

This thesis aims for application of tensegrity systems for the form-finding of cable-stayed bridges. The first section of the thesis briefly reviews history and concept of both, cable-stayed bridges and tensegrity systems. As well it provides main characteristics of tensegrity systems, details their applicability, and presents some of existing examples. The second section of the thesis analyses form-finding methods of tensegrity systems. It suggests an approach of form-finding and writes the computational algorithm. The third one performs form-finding of cable-stayed bridge composed of tensegrity modules. The newly obtained geometry of the considering bridge further analysed with finite element program. The last section of the thesis concludes the results of the study. Furthermore, it provides the findings and recommendations for future research.

Importance of the Thesis

Tensegrity structures being highly efficient for bridging small and mid-distances due to excessive deformations under heavy loading and current lack of analytical tools for proper study and design still stand as top interest of research. They evoke development of new mathematical models, search for computational methods and analytical verifications together with extensive studies all around the world. In connection with previously mentioned aspects this thesis aims to suggest the possibility to apply tensegrity as constructive load carrying system in cable-stayed bridges.

The Main Objective of the Thesis

The main objective is to propose a new geometrical configuration of the cable-stayed bridge by highlighting the state of art of form-finding and applying tensegrity systems.

The Main Tasks of the Thesis

In order to achieve the main objective of the thesis number of particular problems were solved that are described as follows:

1. To review evolution and concept of cable-stayed bridges and tensegrity systems.
2. To review basic tensegrity systems, characteristics and applications of tensegrity systems.
3. To present examples of existing tensegrity structures.
4. To analyse form-finding methods of tensegrity systems.
5. To describe force density method chosen for cable-stayed bridge form-finding procedure.
6. To perform form-finding procedure of cable-stayed bridge by using static extended force density form-finding method composed with multi-paradigm numerical computing environment MATLAB.
7. To design obtained structural configuration of the cable-stayed bridge with nonlinear solver of finite element software SOFiSTiK.

1. CABLE-STAYED BRIDGES AND TENSEGRITY SYSTEMS

In this chapter, an overview of cable-stayed bridges and tensegrity systems is presented. The evolution and concept of cable-stayed bridges are briefly described in Section 1.1 and Section 1.2, respectively. In Section 1.3 evolution of tensegrity systems are stated, and the concept of tensegrity systems are given in Section 1.4. In Section 1.5 examples of basic tensegrity systems are presented. Then the characteristics and applications of tensegrity systems are specified in Section 1.6 and Section 1.7 respectively. Moreover, finally, in Section 1.8 real examples of tensegrity structures are presented.

1.1. Evolution of Cable-Stayed Bridges

Cable-stayed bridge history begins in 1595 when the Croatian inventor Fausto Veranzio introduces the idea of cable-stayed bridges in his book (Fig. 1.1). Later cable stays have been used in suspension bridges structure as a stabilizing element to eliminate the kinematic effects caused by deformation. The first such type bridges were Dryburgh Abbey Bridge near Scottish Borders (1817) (Fig. 1.2) and Victoria Bridge in Bath (1836) (Fig. 1.3), later Albert Bridge in West London (1872) (Fig. 1.4) and Brooklyn Bridge in New York (1883) (Fig. 1.5) were designed and built. Using cable-stayed and suspension bridges combination a very strong bridge can be created, it was revealed by that time designers. So taking advantage of their experience, John A. Roebling designed the Niagara Falls Suspension Bridge (1855) (Fig. 1.6) which connected Ontario and New York and was the longest at that time (Wilson and Gravelle, 1991).

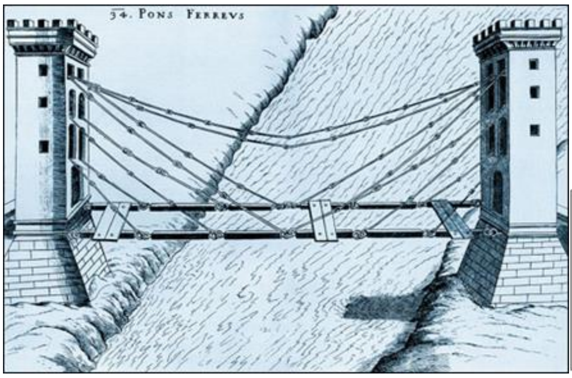

Fig. 1.1. Cable-stayed bridge by Fausto Verenzio, 1595

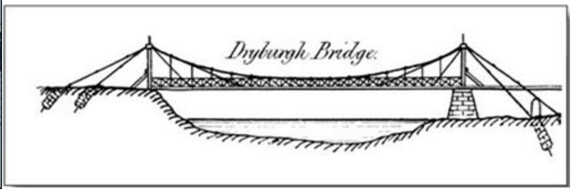

Fig. 1.2. Dryburgh Abbey Bridge near Scottish Borders, 1817

Fig. 1.3. Victoria Bridge in Bath, 1836

Fig. 1.4. Albert Bridge in West London, 1872

The cable-stayed bridge in Bluff Dale, Texas (1890) (Fig. 1.7) is the oldest surviving example of such type of bridges. The early examples of cable-stayed bridges in the twentieth century are Cassagnes bridge in France (1899) (Fig. 1.8) in which the straight part of the cable forces is balanced by a separate horizontal tie wire, preventing significant compression in the deck, and Le Coq Bridge in Lezardrieux, Brittany (1924) (Fig. 1.9). Later cable-stayed aqueduct at Tempul (1926) (Fig. 1.10) and concrete-decked cable-stayed bridge over the Donzère-Mondragon canal in Pierrelatte (1952) (Fig. 1.11) were designed and built. Latter is one of the first modern type bridges but had little effect on the subsequent development of cable-stayed bridges.

Fig. 1.5. Brooklyn Bridge in New York, 1883

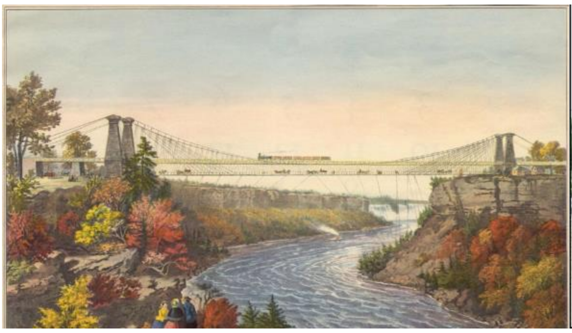

Fig. 1.6. Niagara Falls Suspension Bridge, 1855

Fig. 1.7. Cable-stayed bridge in Bluff Dale, Texas, 1890

Fig. 1.8. Cassagnes bridge in France,1899

Fig. 1.9. Bridge in Lezardrieux, 1924

In the early 1950s, after the war in need of the reconstruction of many bridges over the River Rhine in Germany the concept of the cable-stayed bridge was proposed as the more economical solution for average spans, than the suspension or arch bridge for the same span. The first modern cable-stayed bridge was the Stromsmund Bridge in Sweden (1955) (Fig. 1.12) which was designed by Dischinger. Later the Theodor Heuss Bridge with harp cable arrangement over the River Rhine at Dusseldorf in Germany (1958) (Fig. 1.13) was constructed. The George Street Bridge over the Usk River at Newport, South Wales (1964) (Fig. 1.14) were designed with twin vertical stay planes was the first modern Cable-stayed bridge in the United Kingdom. The first asymmetrical two-span structure was developed in Germany, the Severin Bridge over the Rhine River at Cologne (1961) (Fig. 1.15) which has A-shape pylon and was the first cable-stayed bridge with fan cable arrangement (Parke and Hewson, 2008). The first cable-stayed bridge with semi-fan cable arrangement was the Flehe Bridge in Dusseldorf, Germany over the Rhine River (1979) (Fig. 1.16) which has inverted Y-shape form concrete tower.

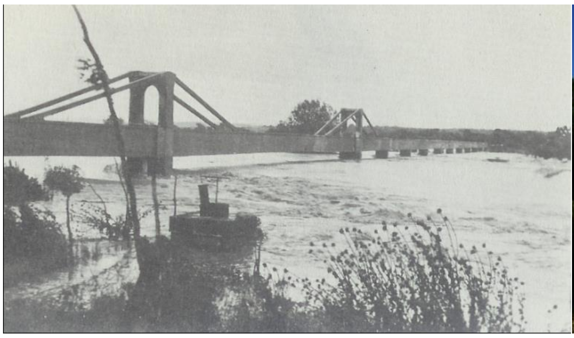

Fig. 1.10. Cable-stayed aqueduct at Tempul, 1926

Fig. 1.11. Cable-stayed bridge in Pierrelatte, 1952

Fig. 1.12. Stromsmund Bridge in Sweden, 1955

Fig. 1.13 Theodor Heuss Bridge in Germany, 1958

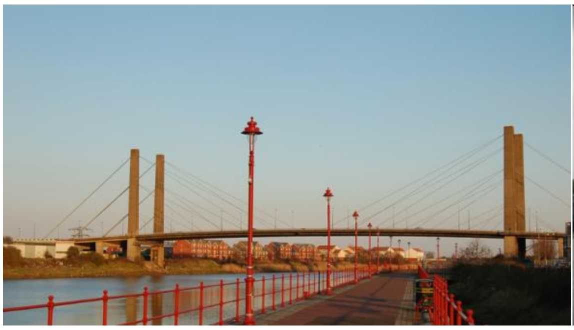

Fig. 1.14. The George Street Bridge in the United Kingdom, 1964

Fig. 1.15. Severin Bridge in Germany, 1961

The economic advantages of cable-stayed bridges are the reason why the concept of supporting deck by cable stays has been preferred as the best solution for a broad range of spans in these days. The Russky Bridge in Vladivostok (Russia) (Fig. 1.17) is the longest span cable-stayed bridge so far completed in 2012 with a main span length of 1104 m. There are two cable-stayed bridges which are under construction with similar main span length, one of them is Hutong Bridge with a main span length of 1092 m and second one Qiongzhou Strait Bridge with a main span length of 1056, both of them are in China.

Fig. 1.16. Flehe Bridge in Germany, 1979

Fig. 1.17. The Russky Bridge in Russia, 2012

1.2. Concept of Cable-Stayed Bridges

The idea of a cable-stayed bridge is straightforward. The deck supports the loads and stays provide medial supports for the bridge so that it can cross a long distance (Tang, 2000). The primary structural elements of cable-stayed bridges are the bridge deck, piers, towers (pylons) and the stays (Calado, 2011). All these elements are under axial forces (Fig. 1.18), with the stays under tension and both the pylon and the deck under compression (Tang, 2000; Calado, 2011).

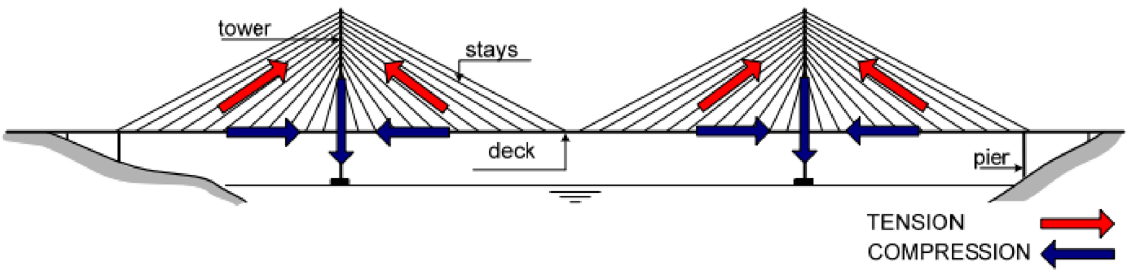

Fig. 1.18. The behavior of a cable-stayed bridge (Calado, 2011)

There is many cable stays arrangement variations in cable-stayed bridges: harp, mono, star, and fan. Parke and Hewson (2008) exclude two basic arrangement for the layout of the stay cables:

- The fan stay system (Fig. 1.19a) (including the semi-fan stay system (Fig. 1.19b)). In the fan stay system cables all connect to or pass over the top of the pylons. In the semi-fan arrangement, the cables terminate near to the top of the tower.
- The harp stay system (Fig. 1.19c). The cables are almost parallel so that the height of their fixing point to the tower is corresponding to the distance from the tower to their installation point on the deck.

In general, axially loaded members are more efficient than flexural members (Tang, 2000). It leads to the use of tensegrity systems in cable-stayed bridges design.

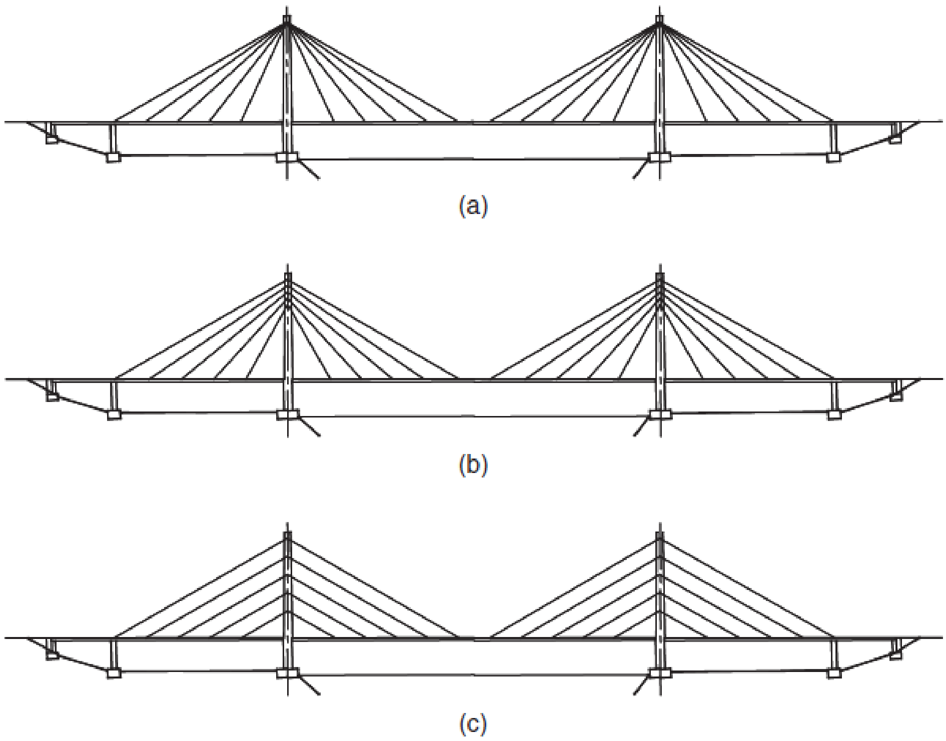

Fig. 1.19. Alternative stay cable arrangements: a) fan stay system; b) semi-fan stay system; c) harp stay system (Parke and Hewson, 2015)

1.3. Evolution of Tensegrity Systems

Even though tensegrity first comprehended in the middle of the twentieth century, signs of tensegrity systems can be seen in sculpture made by Russian constructivist artist Karl Ioganson in 1920 (Motro, 2003). A structure called *gleichgewicht konstruktion* made of three struts, seven tensioned cables, and one cable to change the shape of the structure. In Figure 1.20 graph of Ioganson's construction is presented.

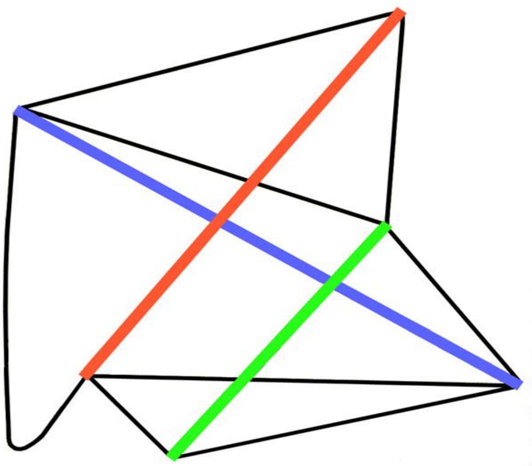

Fig. 1.20. „*Gleichgewicht konstruktion"* by Ioganson, 1920

The origins of tensegrity are arguable, Richard Buckminster Fuller, David Georges Emmerich, and Kenneth D. Snelson considered as inventors of tensegrity (Jauregui, 2004). Each of them granted their patents in almost the same time. Fuller on November 13, 1962, presented U.S. Patent 3,063,521, "Tensile-Integrity Structures". Later Emmerich on September 28, 1964, introduced French Patent No. 1,377,290, "Construction de Reseaux Autotendants and Snelson on February 16, 1965, presented U.S. Patent 3,169,611, "Continuous Tension, Discontinuous Compression Structure". Comparison of three details of the three patents given in Fig. 1.21 below.

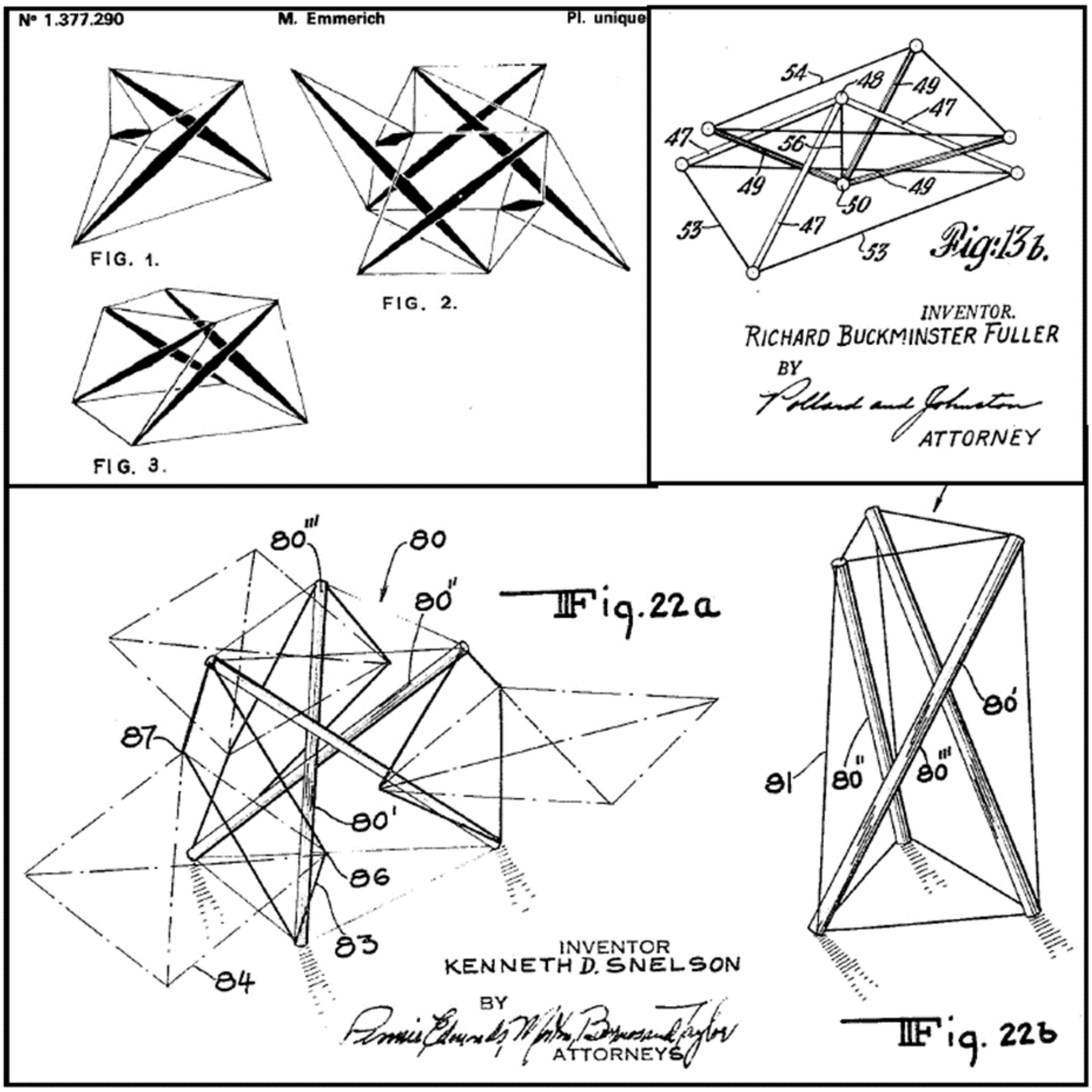

Fig. 1.21. Comparison of three details of the three patents (Jauregui, 2004)

In 1948 K. Snelson influenced by what he had learned from R. B. Fuller in his lectures on geometric models at Black Mountain College and made his X piece (Fig. 1.22) (Jauregui, 2004). After seeing his work, R. B. Fuller realized that it was the answer to a question what he was searching for years (Jauregui, 2004).

Alongside, but separately, David Georges Emmerich started to study new types of structures as tensile prisms and more elaborate tensegrity systems, which he called "structures tendues et autotendants", tensile and self-stressed structures. As a result, he described and patented his "reseaux autotendants", which were the same type of structures that were being studied by Fuller and Snelson.

Nowadays K. Snelson is a well-known artist who designs tensegrities all over the world. One of his structures is the Needle Tower II build in 1969 at the Kroller Müller Museum in Otterlo, the Netherlands (Fig. 1.23). Snelson discovered tensegrity and went the artistic road developing sculptural research whereas Fuller oriented his research more to the analytical side.

Fig. 1.22. „X-column" by Snelson. Illustration donated by the artist

Fig. 1.23. The Needle Tower II by Snelson. Illustration donated by the artist

Fuller and Snelson applied the concept of tensegrity to generate structures that were flexible and firm. Their work is an inspiration to engineers and artists all around the world in the last century and into the present one to design and create tensegrity based structures. In the last 20 years, the broad range of research has been done by a lot of Mathematicians to systematize, categorize, and develop algorithms for tensegrity structures.

1.4. Concept of Tensegrity Systems

Richard Buckminster Fuller proposed tensegrity or tensional integrity term in 1962 (Juan and Tur 2008; Pagitz and Tur, 2009; Rhode-Barbarigos et al., 2010; Amouri et al., 2015; Joshi and Al-Hakkak, 2015). It is a property of structures which are in equilibrium and create an ensemble of tension and compression components. The concept of tensegrity systems has been applied to an extensive range of phenomena from philosophy to cellular mechanics.

Principals of tensegrity structures based on a few simple design patterns. Any of structural member experiences a bending moment because of these patterns. The design patterns are as follows:

- Members are only in pure compression or pure tension if any cable yields or any strut buckles the structure will fail.
- Preload or tensional prestress, which allows cables to be rigid in tension.
- Mechanical stability, which allows the members to remain in tension/compression as stress on the structure increases.

In tensegrity structures, no structural element experiences a bending moment. It makes outstandingly rigid structures for their mass and the cross section of the components.

Several definitions of tensegrities presented below. There is no distinct definition of tensegrities, however all definitions agree that the ensemble of tension and compression components shall be in equilibrium.

Richard Buckminster Fuller in 1962 described tensegrity systems as “islands of compression in an ocean of tension”, later in 1975 he defined tensegrity as follows:

“The word tensegrity is an invention: it is a contraction of tensional integrity. Tensegrity describes a structural-relationship principle in which structural shape is guaranteed by the finitely closed, comprehensively continuous, tensional behaviours of the system and not by the discontinuous and exclusively local compressional member behaviours. Tensegrity provides the ability to yield increasingly without ultimately breaking or coming asunder. The integrity of the whole structure is invested in the finitely closed, tensional-embracement network, and the compressions are local islands.” (Fuller, 1975).

Throughout the same decade, David George Emmerich in 1963 and Kenneth Snelson in 1965 presented similar definitions of tensegrity systems. David George Emmerich definition is:

“Self-stressing structures consist of bars and cables assembled in such a way that the bars remain isolated in a continuum of cables. All these elements must be spaced rigidly and at the same time interlocked by the pre-stressing resulting from the internal stressing of cables without the need for external bearings and anchorage. The whole is maintained firmly like a self-supporting structure, whence the term self-stressing.” (Emmerich, 1963).

Kenneth Snelson definition is:

“Tensegrity describes a closed structural system composed of a set of three or more elongate compression struts within a network of tension tendons, the combined parts mutually supportive in such a way that the struts do not touch one another, but press outwardly against nodal points in the tension network to form a firm, triangulated, prestressed, tension and compression unit.” (Snelson, 1965).

Later in 1976 Anthony Pugh proposed a definition, which is the result of combining the definitions by Richard Buckminster Fuller, David George Emmerich and Kenneth Snelson:

“A tensegrity system is established when a set of discontinuous compression components interacts with a set of continuous tensile components to define a stable volume in space.” (Pugh, 2008).

Rene Motro suggested a current and widely conceptual meaning in 2003:

“A tensegrity is a system in stable self-equilibrated state comprising a discontinuous set of compressed components inside a continuum of tensioned components.” (Motro, 2003).

This definition embeds systems where compressed elements are interconnected as tensegrity structures. In 2001 Robert Skelton proposed Another widely cited definition of a tensegrity systems:

“A Class k tensegrity structure is a stable equilibrium of axially loaded elements, with a maximum of k compressive members connected at the node(s).” (Skelton, 2001).

It should be noted that tensegrity is a comparatively new area of study, currently none of the definitions have been invariably accepted, thus it is very important to know the different meanings in a variety of disciplines and the differences among them.

1.5. Basic Tensegrity Systems

Snelson (2012) has illustrated five essential weave cells as shown in Fig. 1.24 which can be converted into essential tensegrity structures. Among these, the first two geometries on the left-hand side (two and three-way cross units) have add up to triangulation making the tensegrity structures firm. The rest of the geometries are flabby because they have less cables than demanded for full triangulation. By the relative position of cables and complexity of arrangement of struts Pugh (1976) classified four basic paterns of tensegrity structures:

spherical system, star system, irregular system and cylindrical system (Pugh, 1976; Motro, 2003). Due to vast scale application in designing tensegrity structures only spherical systems are reviewed in this work. Spherical systems are divided into rhombic, circuit and zig-zag configurations.

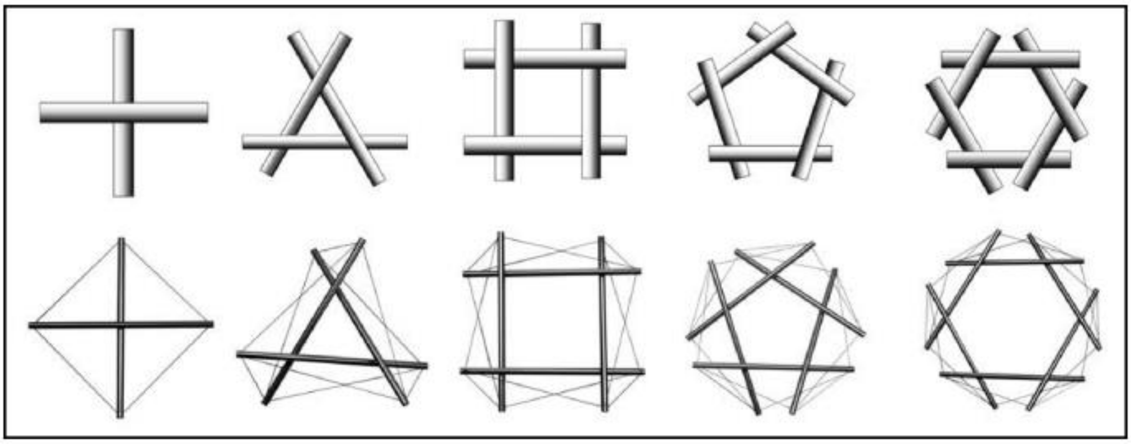

Fig. 1.24. Primary weave cells and equivalent basic tensegrity modules (Snelson, 2012)

1.5.1. Rhombic Configuration

One of the examples of rhombic configuration is so called tensegrity prism or T-prism (see Fig. 1.25). It can be classed as a twisted prism containing two triangular faces twisted on each other (Bansod et al., 2014). The T-prism contains nine cables and three struts. The simple construction of the T-prism makes it one of the most informative members of primary tensegrity structures. To form T-prism, the lengths of one set of cables and struts shall be kept constant while lengths of another set of cables are determined. When one edge of the prism is twisted about the other, the quadrilateral sides of the prism convert to non-planar quadrilaterals. Because of that, two facing angles of each quadrilateral turn into obtuse and acute. To keep structure firm and prestressed the prism is twisted in such a manner that the distance among the obtuse angles is least and in consequence, a fully stable T-prism is formed (Hanaor, 1987; Bansod et al., 2014).

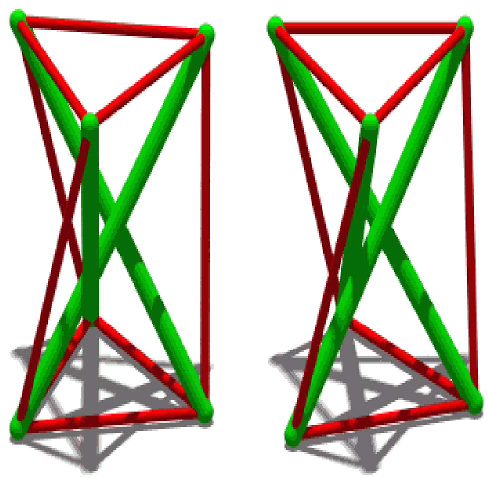

Fig. 1.25. T-prism

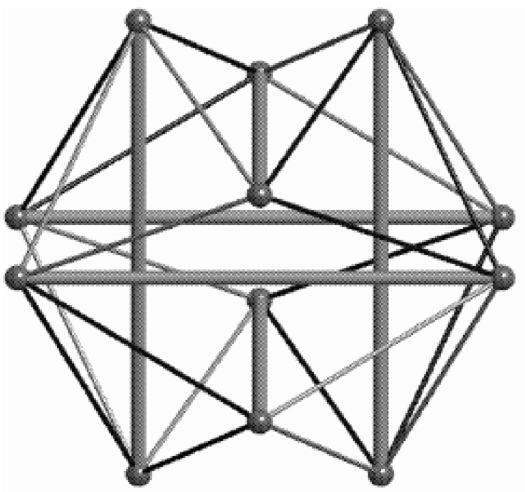

Fig. 1.26. T-icosahedron

Another example of rhombic configuration is so called tensegrity icosahedron or T-icosahedron (see Fig. 1.26). The topology of T-icosahedron describes these tensegrities, every triangle of cables linked to the adjoining one through a strut and two interconnecting cables (Hanaor, 1987). It was first shown by Buckminster Fuller in 1949 and is just one of a small number of tensegrities which show mirror symmetry. This tensegrity belongs to a class of ‘diamond’ type tensegrities as each of its struts is bounded by a diamond form of four cables which are held by two in line struts (Bansod et al., 2014).

More examples of rhombic configuration are presented in Figs. 1.27 and 1.28. T-octahedron presented in Fig. 1.27 consist of three struts and twelve tendons. The three compression struts are not linked to each other as they go at the focal point of the octahedron. They are linked only to their terminals by the inclusive, triangular tension net. T-cuboctahedron is formed by altering the quadrilaterals settled with struts to squares (see Fig. 1.28).

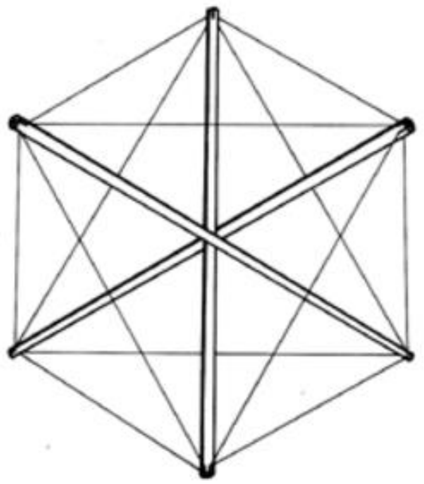

Fig. 1.27. T-octahedron

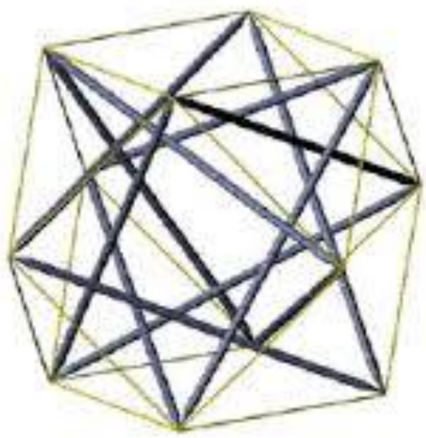

Fig. 1.28. T-cuboctahedron

By opening the octahedral structure from one end and including new layers of struts and tendons and linking both closures of every layer, new tensegrity structures with spherical symmetry can be produced (Bansod et al., 2014).

1.5.2. Circuit Configuration

In circuit configuration systems, the compressed elements are arranged by close circuits which do not meet the standard definition of tensegrity (Hanaor, 1987). Circuit arrangement can be set up by shutting the rhombus made by struts and cables of T-icosahedron (diamond pattern tensegrity)(see Fig. 1.26).

By joining the struts to shape a circuit pattern, the new cables-struts relationship is set up. Circuit design tensegrity structures are concervative in size and capable to resist more prominent external load compared to the tensegrity structure constructed using a diamond design for the similar number of struts. In comparison with a rhombic arrangement with the similar number of struts, circuit arrangement is more rigid (Pugh, 1976). Pugh (1976) in his work described several regular and semiregular polyhedra which can be constructed using this arrangement, for example, T-icosidodecahedron (see Fig. 1.29), T-cuboctahedron (see Fig. 1.30), snub cube, and snub icosahedron, etc. If a different set of two squares are constrained toward each other, then the whole system twists and contracts (Bansod et al., 2014).

As can be visible in Fig. 1. 30, the T-cuboctahedron tensegrity is made of four circuits of three struts each (triangles) and 24 cables characterizing the edges of the T-cuboctahedron (Bansod et al., 2014). The balance of these triangles is guaranteed by four hexagons of cables, each of them being in a plane with one of the triangles and connecting its tops with one top of the other three triangles. In this arrangements struts are linked to each other, and it may be appealed that it is not a genuine tensegrity structure, but since the triangles of struts act as a single compression component, these structures can be viewed as tensegrity structures (Bansod et al., 2014).

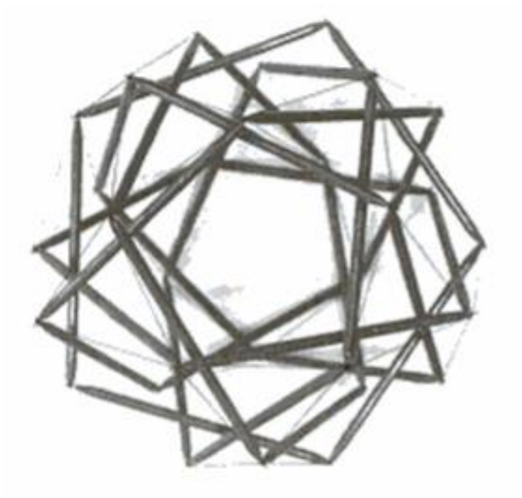

Fig. 1.29. T-icosidodecahedron (Pugh, 1976)

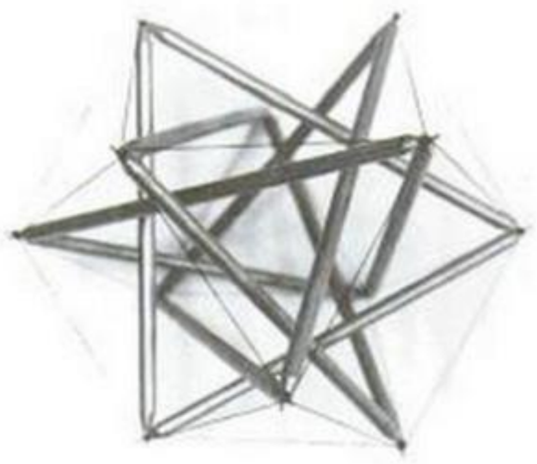

Fig. 1.30. T-cuboctahedron (Pugh, 1976)

T-rhombicuboctahedron has eight triangular and eighteen square faces. The tensegrity structure has twenty-four struts making six square circuits organised in three sets opposite to each other and forty-eight cables (see Fig. 1.31) (Bansod et al., 2014). T-icosidodecahedron contains of thirty struts of equivalent length organised into six non-touching circuits of pentagons and contains sixty cables (see Fig. 1.29) (Bansod et al., 2014).

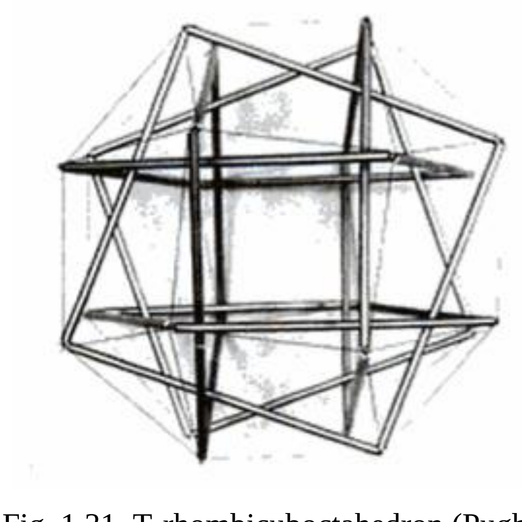

Fig. 1.31. T-rhombicuboctahedron (Pugh, 1976)

1.5.3. Zig Zag Configuration

A ‘zig-zag’ arrangement is gotten from the rhombic arrangement as the initial structure. Both closures of any strut should be linked by three independent cables organized to form a ‘Z’ shape. T-tetrahedron (see Fig. 1.32) is a characteristic example of Z-type arrangement gotten from T-icosahedron (see Fig. 1.26) which categorized to the class of rhombic arrangements.

This sort of arrangement can be achieved by eliminating two cables of the confronting sides of a non-planar rhombus of T-icosahedron. In this manner, the amount of cables are decreased, and their positioning can be changed in a way that three neutral cables are linking both closures of a strut form a ‘Z’ shape (Jauregui, 2004). The T-tetrahedron (see Fig. 1.33) is the zig-zag analog of the diamond T-icosahedron (see Fig. 1.26). Even assuming both structures have six struts, the most imperative change is that T-tetrahedron has four cable triangles,whereas the T-icosahedron has eight of them.

In general, zig-zag structures with Z-type arrangement are more straightforward and less firm because of their lower number of cables than their diamond analogue with rhombic arrangement (Hanaor, 1987).

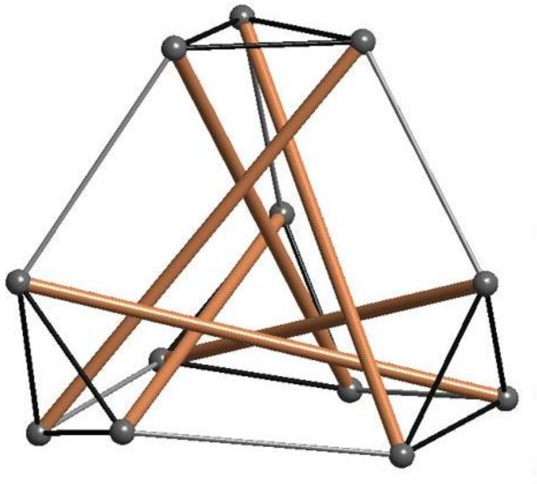

Fig. 1.32. Truncated Tetrahedron (Estrada et al., 2012)

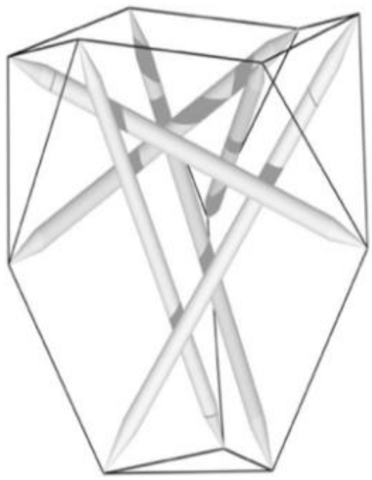

Fig. 1.33. T-Tetrahedron (Hanaor, 1987)

1.6. Characteristics of Tensegrity Systems

Tensegrity as a structural system provides many advantages over conventional structural systems. The most important characteristics of tensegrity systems submitted by Skelton et al. (2001) presented below:

- **Stabilization of the structure through tension**. Under the acting load compressive members loose their stiffness, on the other hand, tensile ones gain it. There is two ways how compressive members loose their stiffness. With on absence of bending moment (forces act exactly through the mass centre of the element) the loss of stiifness invokes the redistribution of the material and increase the mean cross-sectional area on acount of decreased cross-sectional area of tensile members. In the presence of bending moment (forces act not through the mass centre of the element, there is eccentricity) bar becomes slender. For almost all materials, the tensile strength of a longitudinal member is greater than its buckling strength (sand, masonry, and unreinforced concrete misfit this rule). For this reason, a large stiffness-to-mass ratio can be achieved by increasing the use of tensile members.
- **Tensegrity structures are efficient**. The cost-effectiveness of a structure increases with the minimal mass design for a specified set of stiffness characteristics. The unusual arrangement of members in tensegrity structures allows achieving maximum strength with a minimum mass.

- **Tensegrity structures are deployable**. Since the compressive elements of tensegrity structures are either disconnected or connected with pin joints, large displacement, deployability, and storage in a small volume are possible in tensegrity structures. This feature offers an advantages of operation and portability. A transportable bridge or a power transmission tower made as a tensegrity structure could be constructed in the workshop, stored on a truck or helicopter in a small volume, conveyed to the construction site, and positioned using only winches for erection through cable tension. Deployable structures can save transportation costs by reducing required mass or by eliminating assembling human sources.
- **Tensegrity structures are easily tunable**. The same deployment method can also make small adjustments for fine tuning of the loaded structures, or replacement of a damaged structure. Structures that can be tunable will be a major quality of next generation civil engineering structures.
- **Tensegrity structures can be more reliably modelled**. All members are only in pure compression or pure tension. Under an action of external static loads, tensegrity structures bend systematically without bending of individual members. Members that deforms in two or three dimensions are much harder to model than members that deform in only one dimension. For this reason, increased number of tensile members generates more efficient structures.
- **Tensegrity structures can perform multiple functions**. Tensile and compressive members within the tensegrity structures can be multi-functional. At the same time it can be a load-carrying member of the structure, a sensor (measuring tension or length), an actuator (such as nickel-titanium wire), a thermal insulator, or an electrical conductor. Hence, by proper choice of materials and geometry the electrical, thermal, and mechanical energy in material or structure can be controlled.
- **Tensegrity structures have biological basis**. The nanostructure of the spider fibre is a tensegrity structure. It is the strongest natural fibre. If naturally tensegrity is opt for building architecture, then the same incredible efficiency possessed by natural systems can be transferred to human-made systems too.

The advantages of tensegrity over conventional (continuous) structures are listed below:

- Load distributes in a whole structure, because of that there are no critical points of weakness (Kenner, 1976).
- They do not suffer any torsion and buckling due to spacial arrangement and short length of compression members (Fuller, 1975).
- In tensegrity structures forces are transferred naturally and consequently, the members position themselves precisely by aligning with the lines of forces transmitted through the shortest path to withstand the induced stress.
- They are suitable to vibrate and transfer loads very rapidly, and for this reason, tensegrity structures can absorb shocks and seismic vibrations which make them applicable as sensors or actuators (Skelton and Sultan, 1997; Tibert, 2002)
- They can be extended endlessly by adding simple structures.
- Erection of structures using tensegrity concept makes it highly resilient and, at the same time, very economical.

The disadvantages of tensegrity over conventional (continuous) structures are as follows:

- If the structure grows too large, the struts start running into or touching each other (Hanaor, 1987).
- They show comparatively high deflections and low material efficiency as compared to conventional continuous structures (Hanaor, 1987).

- Complex manufacture is a significant barrier for the development of floating compressive structures (Jauregui, 2004).
- There is still a lack of sufficient design tools.
- Within large structures, the resistance of tensegrities is limited to critical load, which is corralated to their dimensions and prestress (Hanaor, 1987).

1.7. Applications of Tensegrity Systems

Tensegrity structures are applicable in a sort of fields like civil engineering, architecture, mechanical engineering, aerospace, and biomechanics (Gilewski et al., 2015; Joshi and Al-Hakkak, 2015). The characteristics of tensegrity structures which make the technology appealing for human use are their deployability and their efficient use of materials. In tensegrity structures, the light tensile elements prevail, while a more material-intensive compressive elements are minimized. For that reason, the construction of buildings, bridges and other structures using tensegrity concept could make them highly durable and very economical at once.

In the domical configuration, tensegrity structures could allow the manufacture of remarkably large structures. When erected within the city limits, these structures could work as frameworks for environmental control, energy transformation and food production. They could be practical in situations where extensive electrical or electromagnetic shielding is needed, or in extra-terrestrial circumstances where micrometeorite preservation is required. Moreover, they could provide the barring or containment of flying animals over large areas, or contain debris from explosions. These domes could encompass vast areas with only minimal support at their perimeters. Suspending structures above the earth on such minimal foundations would allow the suspended structures to escape terrestrial limits in areas where this is useful. For instance of such zones which are crowded or dangerous areas, urban areas, and delicate or rugged territory.

In a spherical arrangement, tensegrity systems could be functional in an outer-space context as superstructures for space stations. Their extreme flexibility makes tensegrity structures capable withstand large structural shocks like earthquakes. Thus, they could be desirable in areas where earthquakes become a problem.

With many research works conducted by civil engineers and architect all over the world, it is now feasible to mathematically model and design tensegrity structures for architectural applications (Joshi and Al-Hakkak, 2015). For this reason, tensegrity bridges and roofs are now a reality.

1.8. Examples of Tensegrity Structures in Civil Engineering

The tensegrity concept gets notable attentiveness among scientists and engineers all over a vast area of disciplines and applications. Towers, large dome structures, stadium roofs, temporarily structures, tents and bridges using tensegrity systems concept have been built or proposed as possible construction type for structures all around the world.

Towers which made of interconnected tensegrity modules are the best-known tensegrity structures. Two of them were designed by Kenneth Snelson: Needle Tower (see Fig. 1.34) and the Needle Tower II (see Fig. 1.35) are an example.

There are some other well know applications of tensegrity concept, such as Munich Olympic Stadium, which has been designed for the Summer Olympics of 1972 (see Fig. 1.36) and Millennium Dome of Richard Rogers (see Fig. 1.37) marking the beginning of the third millennium. The Seoul Olympic Gymnastics Hall (see Fig. 1.38), for the 1988 Summer Olympics and the Georgia Dome (see Fig. 1.39), for the 1996 Summer Olympics, serve as an examples of tensegrity in large structures. A pair of tensegrity skeletons, supporting a membrane roof, has been constructed at Chiba, Japan in 2001.

Fig. 1.34. Needle Tower by Kenneth Snelson, 1968

Fig. 1.35. Needle Tower II by Kenneth Snelson, 1969

Fig. 1.36. Munich Olympic Stadium by Frei Otto, 1972

Fig. 1.37. Millennium Dome by Richard Rogers, 2000

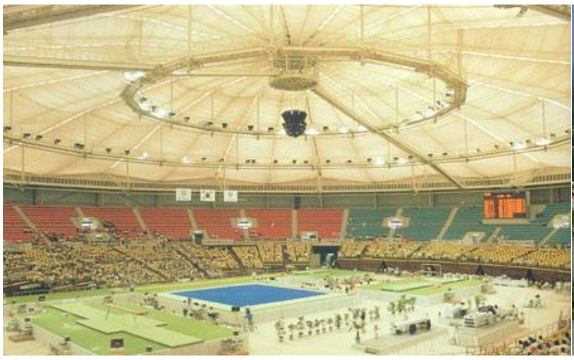

Fig. 1.38. Seoul Olympic Gymnastics Hall

Fig. 1.39. Georgia Dome

An important example of Tensegrity being employed in roof structures is the stadium at La Plata, Argentina (see Fig. 1.40), based on a prize-winning concept developed by architect

Roberto Ferreira. The design modifies the patented Tenstar tensegrity roof concept to the twin peak contour and the plan configuration, and hence, it is more similar to a cable dome structure than to an orthodox roof structure.

Fig. 1.40. The stadium at La Plata, Argentina by Roberto Ferreira

Probably the first tensegrity based bridge structure was proposed in 1996 by Mott MacDonald. It was a proposal for a crossing of the Thames as part of the Millennium Bridge design competition in London. The idea did not win the contest and was never realized. Millennium Bridge proposal by Mott MacDonald in London presented in Fig. 1.41.

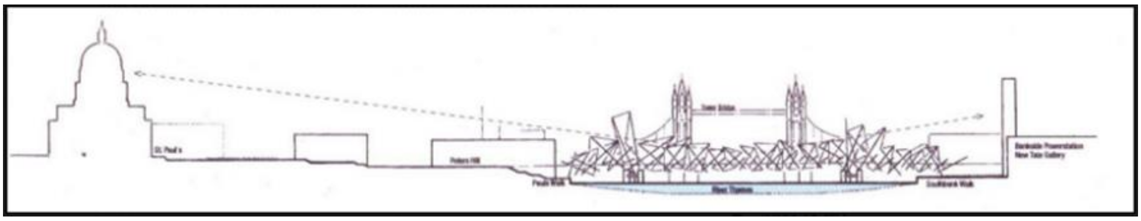

Fig. 1.41. Millennium Bridge proposal in London by Mott MacDonald

In 1998 in Purmerend, Netherlands, architect Jord den Hollander takes a step towards of using tensegrity system in bridge structures. He designed a platform over the water with 18 spans of 4 m, which are suspended by cables to the 36 masts. Mast and deck are not directly connected. Figure 1.42 presents the bridge in Purmerend.

Fig. 1.42. Bridge in Purmerend

Wilkinson Eyre and Arup proposed a 35m footbridge spanning the Washington DC National Museum's Great Hall to connect galleries (see Fig. 1.43). The design was displayed at the Venice Architectural Biennale in 2004. Eyre also proposed a footbridge for the Apraksin Dvor area of St Petersburg. The bridge would be hanged by a related tensegrity cloud (see Fig. 1.44).

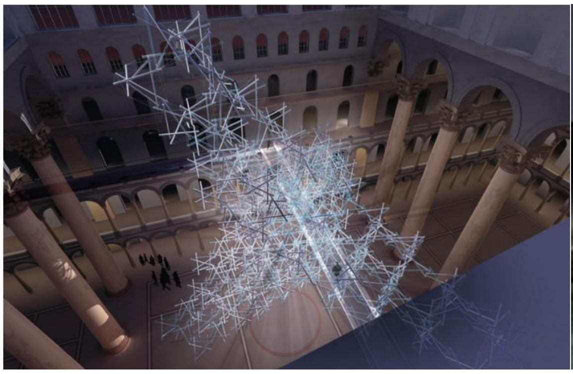

Fig. 1.43. National Building Museum Footbridge Proposal, Washington, DC by Wilkinson Eyre and Arup.

Fig. 1.44. St Petersburg Footbridge Proposal, St. Petersburg, Russia by Wilkinson Eyre

Gomez Jauregui (2004) proposal of advanced use of tensegrity in civil engineering was a bridge composed of several basic modules in a row (see Fig. 1.45). He points out that the structure could be used as a connecting element between two buildings. In the case of existing buildings, the fact that tensegrities form independent structure could be a significant advantage to keeping the existing structure intact without the need of strengthening. In 2005 Andrea Micheletti together with a research group designed a footbridge out of tensegrity module (see Fig. 1.46). Referring the previously mentioned example, Micheletti proposal was based on a row of several basic tensegrity configurations.

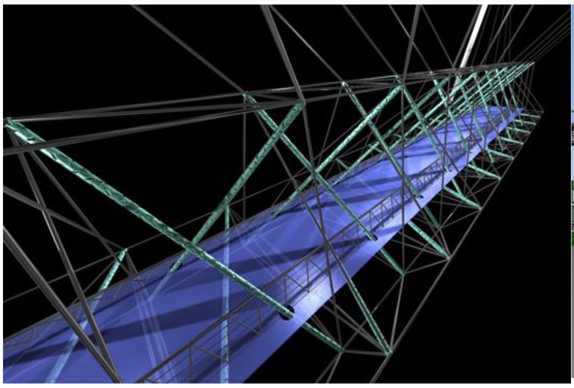

Fig. 1.45. Simplex module footbridge proposal by Jauregui

Fig. 1.46. Footbridge at Tor Vergata

Rhode-Barbarigos et al. (2012) analysed an option to use tensegrity for a deployable pedestrian bridge as part of their work. The design of the bridge composed of four corresponding modules connected base to base creating a total span of 20 m. In Figure 1.47 scheme of the bridge is showed and principle system of deploy process is presented. The different module next to each other creates a tube-like structure.

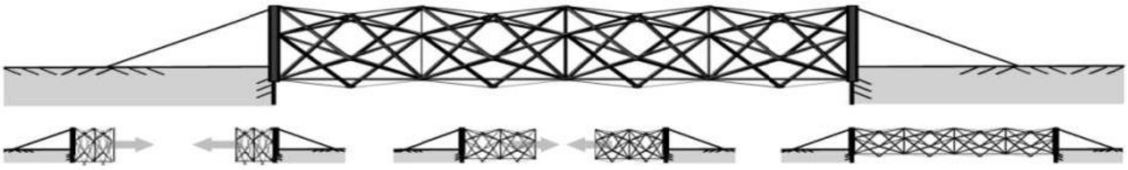

Fig. 1.47. Deployable Hollow Rope Footbridge

The most recent proposal is the Suspended Tensegrity Bridge design by Stefano Paradiso and Marco Mucedola (see Fig. 1.48). The project covers a footbridge over the Sesia river, not far from Greggio.

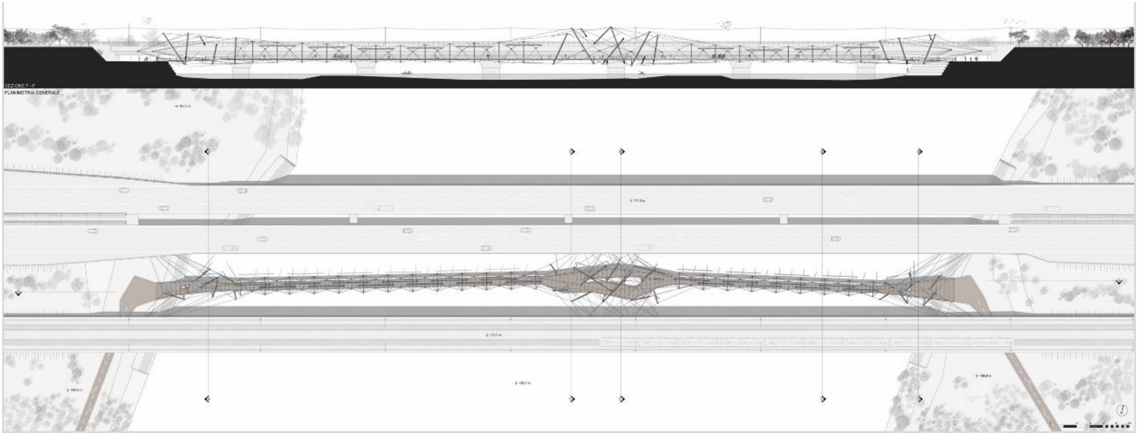

Fig. 1.48. Suspended Tensegrity Bridge by Stefano Paradiso and Marco Mucedola

The most distinguished tensegrity bridge ever build is the Kurilpa Bridge in Brisbane, Australia (see Fig. 1.49) which was opened for pedestrian and cyclist in 2009. Designed by Cox Rayner Architects the Kurilpa Bridge is 470 m long. Struts and cables floating above and beside the deck attain these condemnatory purposes: suspends the sunroof, prevent sideways buckling and withstand lateral forces from the wind, earthquakes and patch loads on the floor.

Fig. 1.49. Kurilpa Bridge in Brisbane, Australia

1.9. Concluding Remarks of Chapter 1

Chapter 1 draws following key findings:

1. The evolution and concept of cable-stayed and tensegrity systems were presented. The idea of a cable-stayed bridge is straightforward. The deck supports the loads and stays provide medial supports for the bridge so that it can cross a long

distance. However, tensegrity system may be associate with a discrete case of truss which composes structural members having particular functions. All the members within tensegrity structure are axially loaded and none of the individual's experience any of bending moments.

2. When considering mass consumption tensegrity systems provide better capability to resist external loads in comparison to conventional structures. They are lightweight, adjustable and mass efficient. Within tensegrity structures, longitudinal members have uncommon non-orthogonal arrangement, which enables to achieve maximum mass efficiency by introducing maximum strength with minimal mass. This property of tensegrity structures may possibly alternate to distinctive rigid structures. The concept of tensegrity presents new opportunities for structural expression in civil engineering.
3. Few, but successful arrangements of tensegrity systems by means of cable-stayed bridges, domes and masts show that despite artificial structural basis tensegrity can serve as highly structurally efficient and sustainable supporting system.
4. Tensegrity structures experience a highly efficient structural behavior reasoned by exceptionally axial resistance, structural control performed by the pretension of tensile members and easier predictable systematic instability.
5. The extreme flexibility of tensegrity structures makes them capable of withstanding large structural shocks and determines applicability for structures built in the areas of natural hazards.
6. Due to wide variation in structural configuration tensegrity structures serve as a desirable tool for architects in order to satisfy strict aesthetic requirements and realize their self-expression; none the less they are desirable by structural engineers as a problem of form-finding.

2. FORM-FINDING

In this chapter, an overview of form-finding methods of tensegrity structures is given, and algorithm for form-finding of tensegrity bridges is proposed. Section 2,1 describes the definition of form-finding, while Section 2.2 provides and categorizes the existing form-finding methods of tensegrity structures. Section 2.3 describes the methodology of form-finding of tensegrity bridge using extended force density method (EFDM)

2.1. Definition

Form-finding or shape-finding is the design process in which the shape of the structure is set on. Several definitions of form-finding are presented below.

As stated by Bletzinger et al. definition of form-finding is (Veenendaal and Block, 2012; Luczkowski et al., 2016):

Finding a shape of equilibrium of forces in a given boundary with respect to a certain stress state.

Modern form-finding definitions were introduced by Coenders and Bosia in 2006 and by Basso and Del Grosso in 2011 (Veenendaal and Block, 2012; Luczkowski et al., 2016). Coenders and Bosia definition is:

Finding an appropriate architectural and structural shape.

Basso and Del Grosso definition is:

A structural optimisation process which uses the nodal coordinates as variables.

Basso and Del Grosso definition is much wider and shows that contemporary object of interest is not only cable structures and membranes roofs like in the past, but also skyscrapers and long-span bridges (Luczkowski et al., 2016).

2.2. Form Finding Methods of Tensegrity Structures

In this section, a study of the existing form-finding methods for tensegrity structures and their categorization is done to set a right method for an outline of statically stable tensegrities. From 1960's to 2010's various methods of form-finding have been established. There are two main groups of existing form-finding methods for tensegrity structures: kinematical methods and statical methods (Juan and Tur, 2008; Pagitz and Tur, 2009; Tibert and Pellegrino, 20011).

The former methods are outlined by increasing (decreasing) the length of the struts (cables) and keeping the length of the cables (struts) constant until a maximum (minimum) is reached (Juan and Tur, 2008; Pagitz and Tur, 2009; Tibert and Pellegrino, 20011). In latter methods, when topology and forces of members are given, the relationship is set up between equilibrium configurations of a structure (Juan and Tur, 2008; Pagitz and Tur, 2009; Tibert and Pellegrino, 20011).

2.2.1. Kinematical Methods

The analytical solution (AS) was proposed by Connelly and Terrell in 1995. It embeds the coordinates of each node as a function of geometric parameters and then performs the maximization (minimization) of the length expressions of struts (cables) with a starting point refereeing to arbitrary configuration (Juan and Tur, 2008; Tibert and Pellegrino, 2011). This method is easily applicable for the symmetric structures, but due to extensive number of variables it becomes unpractical for non-symmetric tensegrities. Research in this field also has been done by Li et al. (2010) and Koohestan and Guest (2013).

Pellegrino (1986) and Burkhardt (2006) proposed non-linear programming (NLP) technique by turning the form-finding method of tensegrity structures into another one of constrained minimization problem (Juan and Tur, 2008; Tibert and Pellegrino, 2011). For this approach, a valid initial configuration is needed to start with and then try to minimize (maximize) the length of same struts (cables), but they do not consider any stress restriction (Juan and Tur 2008). For that reason, even being geometrically correct, the outcome structure may not be firm. Research in this field also has been done by Burkhardt (2006).

Finally, Motro (1984) and Belkacem (1987) suggested the dynamic relaxation (DR) method for tensegrity structures (Juan and Tur, 2008; Tibert and Pellegrino, 2011). This method solves a fictitious dynamic model regarding the acceleration, velocity, and displacement by considering initial configuration in order to get the equilibrate state. DR is only practical for small size structures, structures, it considers equilibrium state and external forces though (Juan and Tur 2008). Research in this field also has been done by Zhang et al. (2006), Ali et al. (2010) and Luczkowski et al. (2016).

2.2.2. Statical Methods

The AS was proposed by Kenner (1976), where he applied node equilibrium conditions and symmetry arguments to come up with the stable configurations of tensegrity structures (Juan and Tur, 2008; Tibert and Pellegrino, 2011). This method guarantees the stability of the structure without any external load (Juan and Tur 2008).

Schek (1974) and Linkwitz (1999) suggested the force density method (FDM), where nonlinear equilibrium equations are transformed into linear equations (Tibert and Pellegrino, 2003; Juan and Tur, 2008). This approach demands the understanding of the stress coefficients for all members who are a considerable disadvantage since some combinations of stresses may not have real applications in specified space (Juan and Tur 2008). There is no possibility to control the length of the members which is another disadvantage of this method. Although cons this approach is the most used for form finding of tensegrity structures. There are some modifications of this approach which eliminates the above described problems. Research in this field also has been done by: Schek (1973), Hernandez-Montes et al. (2006), Zhang and Ohsaki (2006), Lee et al. (2009), Tran and Lee (2009), Xu and Luo (2009), Miki and Kawaguchi (2010), Tran and Lee (2010), Quagliaroli (2011), Lee (2012), Veenendaal and Block (2012), Quagliaroli and Malerba (2013), Aboul-Nasr and Mourad (2015), Cercadillo-Garcia and Fernandez-Cabo (2016), Harichandran and Sreevalli (2016), Lee and Lee (2016), Luczkowski et al. (2016).

Energy based form-finding method (EM) was presented by Connelly (1993). In this method, he set an energy function to a tensegrity and searches the minimum of this function, which is sets to test the positive semi-definiteness of the stress matrix (Tibert and Pellegrino, 2003; Juan and Tur, 2008). This stress matrix is analogous to the force density matrix used in the force density method (Juan and Tur, 2008; Tibert and Pellegrino, 2011).

Reduced coordinates method (RCM) was introduced by Sultan et al. (1999). He considered a set of generalized coordinates and used symbolic manipulation to get the equilibrium matrix (Juan and Tur, 2008; Tibert and Pellegrino, 2011). Due to the size of the equilibrium matrix, the solution of this problem is complex (Juan and Tur, 2008). Research in this field also has been done by Juan and Tur (2008).

Micheletti and Williams (2004) in their method solve a system of differential equations (DE) to get stable configuration by changing the length of a given edge and finding the change in length of the other sides (Juan and Tur, 2008).

Masic et al. (2005) developed a modified version of the force density method, the algebraic form-finding method (AM). In this method, they included shape constraints and considered how the regularity properties can be used to efficiently reduce the number of force density variables, equilibrium equations and geometrical variables (Juan and Tur 2008).

2.2.3. Other Methods

Successive approximation method (SAM) was presented by Zhang et al. (2006). In this approach first is found the set of axial forces and then the corresponding nodal coordinates, equilibrium conditions and structural constraints (Juan and Tur, 2008). This method mainly obtains the bases for the self-stress and positioning subspaces and then requires to fix some stresses and coordinates equal to the dimension of those subspaces, respectively, in order to find the final solution (Juan and Tur, 2008).

Paul et al. (2005) presented a different approach based on genetic algorithms (GA) to find the topology which assures stability. In which genetic algorithms are used to evaluate an initial arbitrary topology into a stable one in the workspace (Juan and Tur 2008). This method is fit to generate asymmetrical structures. Research in this field also has been done by Chisari et al. (2015), Gan et al. (2015), Yamamoto et al. 2011, Koohestani (2012), Skelton et al. (2013).

Masic et al. (2006) using non-linear programming techniques introduced by Pellegrino (1986), developed a method which searches for the topology, geometry, and pre-stress of a structure under external forces, and taking into account strength, buckling and form constraints (Juan and Tur 2008). The application of sequential quadratic programming (SQP) allows the algorithm to find stable configurations starting from a random one. Some uniqueness is in the gradient function due to the use of the element length may source the algorithm to diverge or as an alternative, link up to a non-optimum solution (Juan and Tur 2008). Also, their form-finding method holds some physical phenomena due to external loads (Juan and Tur 2008).

Finally, Estrada et al. (2006) introduced a numerical method (NM) which only requires information about the type of each edge and the topology, but does not account for external forces (Juan and Tur, 2008). Using rank constraints the equilibrium geometry and force densities for each edge are iteratively calculated on both stress and rigidity matrix (Juan and Tur, 2008). Research in this field also has been done by Koohestani and Guest (2013), Cheong et al. (2014).

2.2.4. Summary of Form-Finding Methods

Table 2.1 summarizes previously discussed form-finding methods.

Table 2.1. Summary of form-finding methods (Juan and Tur, 2008)

Method name	Class	Assures stability	Needs a valid initial configuration	Uses symmetry	Needs an initial topology	Uses external forces
AS	Kinematic	No	No	Yes	Yes	No
N	Kinematic	No	Yes	No	Yes	No
DR	Kinematic	Yes	Yes	No	Yes	Yes
AS	Static	Yes	No	Yes	Yes	No
FDM	Static	Given	No	No	Yes	Yes
EM	Static	Yes	No	No	Yes	No
RCM	Static	Yes	No	Yes	Yes	No
DE	Static	Yes	Yes	No	Yes	No
SAM	Both	Some stresses have to be fixed	Some coordinates have to be fixed	No	Yes	No
AM	Static	Given	No	No	Yes	Yes
GA	Topologic	Yes	No	No	No	No
SQP	Both	Yes	No	No	No	Yes
NM	Both	Yes	No	No	Yes	No

2.3. Force Density Method for Form-Finding of Tensegrity Bridge

Through last years there has been a growing development of form-finding techniques, also addressed to the study of self-stressed systems. Among the others, force density method is a flexible approach, which acts on tensegrity structures without no need of assumptions regarding geometry or material properties. This section focuses on extending this approach in order to apply it for form-finding of cable-stayed bridge's systems. New proposal provides the possibility to define the boundary conditions referring initial fixed nodal reactions or, in other words, to alter the positions of a definite number of nodes and, at the same time, to inflict the intensity of the reaction force. A new form-finding technique enables to determine the distinct equilibrium shape (geometry and associated prestress) of a system composed of cables and struts under arbitrary loading conditions. This chapter presents the force density method based on researches performed by Schek (1974), Quagliaroli (2011), Quagliaroli and Malerba (2013), Aboul-Nasr and Mourad (2015).

2.3.1. Force Density Method

Considering a general tensegrity structure, having n free nodes and n_f fixed nodes (the total number of nodes is $n_s = n + n_f$), connected by m members, it is assumed that:

- The structure made of straight cable and strut elements joined at the nodes. Part of the nodes is free, part of them is fixed.
- The structure connectivity is known, and the nodal coordinates define its geometry.
- The cable and strut elements are weightless.
- The structure is subjected to concentrated forces, applied at the nodes.

Concerning the ith node of Fig. 2.1, the equilibrium equations in the x, y, z directions are as follows:

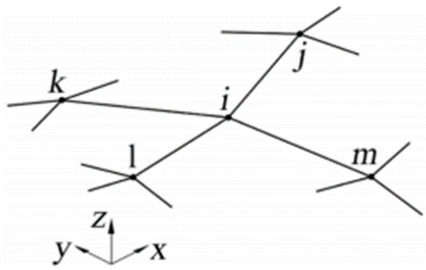

Fig. 2.1. Generic free node (Quagliaroli and Malerba, 2013)

$$\begin{cases} T_{ij}\dfrac{x_j - x_i}{L_{ij}} + T_{ik}\dfrac{x_k - x_i}{L_{ik}} + T_{il}\dfrac{x_l - x_i}{L_{il}} + T_{im}\dfrac{x_m - x_i}{L_{im}} + F_{xi} = 0 \\ T_{ij}\dfrac{y_j - y_i}{L_{ij}} + T_{ik}\dfrac{y_k - y_i}{L_{ik}} + T_{il}\dfrac{y_l - y_i}{L_{il}} + T_{im}\dfrac{y_m - y_i}{L_{im}} + F_{yi} = 0 \\ T_{ij}\dfrac{z_j - z_i}{L_{ij}} + T_{ik}\dfrac{z_k - z_i}{L_{ik}} + T_{il}\dfrac{z_l - z_i}{L_{il}} + T_{im}\dfrac{z_m - z_i}{L_{im}} + F_{zi} = 0 \end{cases} \quad (2.1)$$

where

T_{ij} – the tensile (compressive) force of the cable (strut) element between the nodes i and j

L_{ij} – the length of the cable (strut) element between the nodes i and j (Eq. (2.2))

x_i, y_i, z_i – node coordinates in x, y, z directions

F_{xi}, F_{yi} F_{zi} – nodal forces in x, y, z directions

$$L_{ij} = \sqrt{\left(x_i - x_j\right)^2 + \left(y_i - y_j\right)^2 + \left(z_i - z_j\right)^2} \tag{2.2}$$

To set Eq. (2.1) into matrix form the following vectors and matrices introduced:

- $\boldsymbol{x}$, $\boldsymbol{y}$, $\boldsymbol{z}$, $[n \times 1]$, coordinates of the free nodes.
- $\boldsymbol{x}_f$, $\boldsymbol{y}_f$, $\boldsymbol{z}_f$, $[n_f \times 1]$, coordinates of the fixed nodes.
- $\boldsymbol{f}_x$, $\boldsymbol{f}_y$, $\boldsymbol{f}_z$, $[n \times 1]$, nodal forces.
- $\boldsymbol{l}$, $[m \times 1]$, length of the elements.
- $\boldsymbol{t}$, $[m \times 1]$, axial forces in the elements.

The first step in the force density method is to define the network connectivity. For a structure with m members, n free nodes and n_f fixed nodes, its topology can be described by a connectivity matrix $\boldsymbol{C}_s$, having dimensions $[m \times n_s]$. If member k connects nodes i and j ($i < j$), then the ith and jth elements of the kth row of $\boldsymbol{C}_s$ are set to 1 and -1, respectively, as:

$$\boldsymbol{C}_{s(k,p)} = \begin{cases} 1 & for\ p = i \\ -1 & for\ p = j \\ 0 & for\ other\ cases. \end{cases} \tag{2.3}$$

If free nodes numbered first, $\boldsymbol{C}_s$ can be separated as:

$$\boldsymbol{C}_s = \left[\boldsymbol{C}\ \boldsymbol{C}_f\right] \tag{2.4}$$

where

$\boldsymbol{C}$ – connectivity matrix of free nodes

$\boldsymbol{C}_f$ – connectivity matrix of fixed nodes

The difference between the pairs of coordinates in the three directions x, y, z are:

$$\boldsymbol{u} = \boldsymbol{C}_s \boldsymbol{x}_s = \boldsymbol{C}\boldsymbol{x} + \boldsymbol{C}_f \boldsymbol{x}_f \tag{2.5}$$

$$\boldsymbol{v} = \boldsymbol{C}_s \boldsymbol{y}_s = \boldsymbol{C}\boldsymbol{y} + \boldsymbol{C}_f \boldsymbol{y}_f \tag{2.6}$$

$$\boldsymbol{w} = \boldsymbol{C}_s \boldsymbol{z}_s = \boldsymbol{C}\boldsymbol{z} + \boldsymbol{C}_f \boldsymbol{z}_f \tag{2.7}$$

The free nodes equilibrium equation expressed as:

$$\begin{bmatrix} \boldsymbol{C}^T \boldsymbol{U} \boldsymbol{L}^{-1} \\ \boldsymbol{C}^T \boldsymbol{V} \boldsymbol{L}^{-1} \\ \boldsymbol{C}^T \boldsymbol{W} \boldsymbol{L}^{-1} \end{bmatrix} \boldsymbol{t} = \begin{bmatrix} \boldsymbol{f}_x \\ \boldsymbol{f}_y \\ \boldsymbol{f}_z \end{bmatrix} \rightarrow \boldsymbol{A}\boldsymbol{t} = \boldsymbol{f} \tag{2.8}$$

where

$\boldsymbol{U}$ – diagonal matrix of vector $\boldsymbol{u}$; $\boldsymbol{U} = diag(\boldsymbol{u})$

$\boldsymbol{V}$ – diagonal matrix of vector $\boldsymbol{v}$; $\boldsymbol{V} = diag(\boldsymbol{v})$

$\boldsymbol{W}$ – diagonal matrix of vector $\boldsymbol{w}$; $\boldsymbol{W} = diag(\boldsymbol{w})$

$\boldsymbol{L}$ – diagonal matrix of vector $\boldsymbol{l}$; $\boldsymbol{L} = diag(\boldsymbol{l})$

$\boldsymbol{A}$ – equilibrium matrix of free nodes

Force density coefficient is defined as ratio of its member force to its member length $q_{ij} = T_{ij}/L_{ij}$, moreover, the force density vector consisting of force densities of all members is calculated by:

$$\boldsymbol{q} = \boldsymbol{L}^{-1}\boldsymbol{t} \tag{2.9}$$

where
$\boldsymbol{q}$ – force density vector

The equations of the system (2.8) became linear and uncoupled in the three directions x, y, z:

$$\boldsymbol{C}^T\boldsymbol{U}\boldsymbol{q} = \boldsymbol{f}_x \tag{2.10}$$

$$\boldsymbol{C}^T\boldsymbol{V}\boldsymbol{q} = \boldsymbol{f}_y \tag{2.11}$$

$$\boldsymbol{C}^T\boldsymbol{W}\boldsymbol{q} = \boldsymbol{f}_z \tag{2.12}$$

By introducing the diagonal matrix $\boldsymbol{Q} = diag(\boldsymbol{q})$ and substituting $\boldsymbol{u}$, $\boldsymbol{v}$, $\boldsymbol{w}$ as given by Eq. (2.5), (2.6) and (2.7), it becomes:

$$\boldsymbol{C}^T\boldsymbol{Q}\boldsymbol{u} = (\boldsymbol{C}^T\boldsymbol{Q}\boldsymbol{C})\boldsymbol{x} + \left(\boldsymbol{C}^T\boldsymbol{Q}\boldsymbol{C}_f\right)\boldsymbol{x}_f = \boldsymbol{f}_x \tag{2.13}$$

$$\boldsymbol{C}^T\boldsymbol{Q}\boldsymbol{v} = (\boldsymbol{C}^T\boldsymbol{Q}\boldsymbol{C})\boldsymbol{y} + \left(\boldsymbol{C}^T\boldsymbol{Q}\boldsymbol{C}_f\right)\boldsymbol{y}_f = \boldsymbol{f}_y \tag{2.14}$$

$$\boldsymbol{C}^T\boldsymbol{Q}\boldsymbol{w} = (\boldsymbol{C}^T\boldsymbol{Q}\boldsymbol{C})\boldsymbol{z} + \left(\boldsymbol{C}^T\boldsymbol{Q}\boldsymbol{C}_f\right)\boldsymbol{z}_f = \boldsymbol{f}_z \tag{2.15}$$

The Eq. (2.13), (2.14) and (2.15) can be rewritten as:

$$\boldsymbol{D}\boldsymbol{x} = \boldsymbol{f}_x - \boldsymbol{D}_f\boldsymbol{x}_f \tag{2.16}$$

$$\boldsymbol{D}\boldsymbol{y} = \boldsymbol{f}_y - \boldsymbol{D}_f\boldsymbol{y}_f \tag{2.17}$$

$$\boldsymbol{D}\boldsymbol{z} = \boldsymbol{f}_z - \boldsymbol{D}_f\boldsymbol{z}_f \tag{2.18}$$

where
$\boldsymbol{D}$ – force density matrix of the free nodes (Eq. (2.19))
$\boldsymbol{D}_f$ – force density matrix of the fixed nodes (Eq. (2.20))

$$\boldsymbol{D} = \boldsymbol{C}^T\boldsymbol{Q}\boldsymbol{C} \tag{2.19}$$

$$\boldsymbol{D}_f = \boldsymbol{C}^T\boldsymbol{Q}\boldsymbol{C}_f \tag{2.20}$$

Then the solutions of Eq. (2.16), (2.17) and (2.18) are:

$$\boldsymbol{x} = \boldsymbol{D}^{-1}\left(\boldsymbol{f}_x - \boldsymbol{D}_f\boldsymbol{x}_f\right) \tag{2.21}$$

$$\boldsymbol{y} = \boldsymbol{D}^{-1}\left(\boldsymbol{f}_y - \boldsymbol{D}_f\boldsymbol{y}_f\right) \tag{2.22}$$

$$\boldsymbol{z} = \boldsymbol{D}^{-1}\left(\boldsymbol{f}_z - \boldsymbol{D}_f\boldsymbol{z}_f\right) \tag{2.23}$$

Given topology and assumed vector $\boldsymbol{q}$ of force densities, Eq. (2.21), (2.22), and (2.23) allows to find the unique equilibrium configuration of the system.

2.3.1.1. General Constraint

The linear expression of the force density method presented above makes it feasible to find all the possible equilibrium arrangements of a cable strut system with a certainly given connectivity and with given boundary conditions on the nodes. Each unique configuration corresponds to an assumed force density distribution. The possibility of imposing assigned relative distance among the nodes, the tensile level in the elements and their initial length, was introduced by Schek (1974).

By assuming that all these conditions are the function of the nodal coordinates and the force densities, the additional generic state obtains the following form:

$$g_i(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{z}, \boldsymbol{q}) = 0 \quad (i = 1{:}\, r; r < m) \tag{2.24}$$

For all the r conditions:

$$\boldsymbol{g}(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{z}, \boldsymbol{q}) = \boldsymbol{g}(\boldsymbol{x}(\boldsymbol{q}), \boldsymbol{y}(\boldsymbol{q}), \boldsymbol{z}(\boldsymbol{q}), \boldsymbol{q}) = \boldsymbol{0} \tag{2.25}$$

The initial force density vector has been set to $\boldsymbol{q}^{(0)}$. For this assumed force density state, Eq. (2.25) generally is not satisfied, search of new vector is required:

$$\boldsymbol{q}^{(1)} = \boldsymbol{q}^{(0)} + \Delta\boldsymbol{q} \tag{2.26}$$

The solution searched in an iterative form. The Newton method has been selected in order to find the vector $\Delta\boldsymbol{q}$ which fulfills the following linearization condition:

$$\boldsymbol{g}\left(\boldsymbol{q}^{(0)}\right) + \frac{\partial \boldsymbol{g}\left(\boldsymbol{q}^{(0)}\right)}{\partial \boldsymbol{q}} \Delta\boldsymbol{q} = 0 \tag{2.27}$$

By using Jacobian matrix $\boldsymbol{G}^T$ and misfit $\boldsymbol{r}$:

$$\boldsymbol{G}^T = \frac{\partial \boldsymbol{g}\left(\boldsymbol{q}^{(0)}\right)}{\partial \boldsymbol{q}} \tag{2.28}$$

$$\boldsymbol{r} = -\boldsymbol{g}\left(\boldsymbol{q}^{(0)}\right) \tag{2.29}$$

Equation (2.27) becomes:

$$\boldsymbol{G}^T \Delta\boldsymbol{q} = \boldsymbol{r} \tag{2.30}$$

In this way, the linear system has been obtained, which coefficient matrix has dimensions $[r \times m]$. In a form-finding problem, the number of the added conditions r cannot be larger than the number of the free parameters, which equals the number of the members of the structure m.

Being $m > r$, the system (2.30) is underdetermined and states ∞^{m-r} solutions. The minimum norm should be found among the infinite solutions. In other words, among all the vectors which satisfy the system (2.30) there should be found the solution $\Delta\boldsymbol{q}$ which as well satisfies the equation:

$$\Delta \boldsymbol{q} = argmin\|\Delta \boldsymbol{q}\|_2^2 \tag{2.31}$$

Equations (2.30) and (2.31) form a problem of constrained optimisation, seeking for the minimum of the function:

$$f(\Delta \boldsymbol{q}) = \Delta \boldsymbol{q}^T \Delta \boldsymbol{q}, \qquad \text{with the constraints } \boldsymbol{G}^T \Delta \boldsymbol{q} = \boldsymbol{r} \tag{2.32}$$

By pertaining the Lagrange multipliers method, it becomes:

$$\Delta \boldsymbol{q} = \boldsymbol{G}(\boldsymbol{G}^T \boldsymbol{G})^{-1} \boldsymbol{r} \tag{2.33}$$

Being the initial conditions approximated via the linearization given by Eq. (2.27), the solution attained in an iterative way. At the set up of each iteration it has been assumed that:

$$\boldsymbol{q}^{(k+1)} = \boldsymbol{q}^{(k)} + \Delta \boldsymbol{q}^{(k)} \tag{2.34}$$

Then, by updating the corresponding matrix $\boldsymbol{G}^T$ and vector $\boldsymbol{r}$ and, Eq. (2.33) determines the vector $\Delta \boldsymbol{q}$. The iterative process stops, when the following condition results in small tolerance:

$$\boldsymbol{g}\big(\boldsymbol{q}^{(k)}\big) = -\boldsymbol{r}\big(\boldsymbol{q}^{(k)}\big) = \boldsymbol{0} \tag{2.35}$$

2.3.1.2. Jacobian Matrix

The iterative approach needs an effective formulation of the Jacobian matrix $\boldsymbol{G}^T$. The extension of $\boldsymbol{G}^T$ by using chain rule derives:

$$\boldsymbol{G}^T = \frac{\partial \boldsymbol{g}}{\partial \boldsymbol{q}} = \frac{\partial \boldsymbol{g}}{\partial \boldsymbol{x}}\frac{\partial \boldsymbol{x}}{\partial \boldsymbol{q}} + \frac{\partial \boldsymbol{g}}{\partial \boldsymbol{y}}\frac{\partial \boldsymbol{y}}{\partial \boldsymbol{q}} + \frac{\partial \boldsymbol{g}}{\partial \boldsymbol{z}}\frac{\partial \boldsymbol{z}}{\partial \boldsymbol{q}} + \frac{\partial \boldsymbol{g}}{\partial \boldsymbol{q}} \tag{2.36}$$

The derivatives $\partial \boldsymbol{x}/\partial \boldsymbol{q}$, $\partial \boldsymbol{y}/\partial \boldsymbol{q}$, $\partial \boldsymbol{z}/\partial \boldsymbol{q}$ are independent of Eq. (2.25) and can be expressed considering known quantities in this way:

$$\frac{\partial \boldsymbol{x}}{\partial \boldsymbol{q}} = -\boldsymbol{D}^{-1}\boldsymbol{C}^T\boldsymbol{U} \tag{2.37}$$

$$\frac{\partial \boldsymbol{y}}{\partial \boldsymbol{q}} = -\boldsymbol{D}^{-1}\boldsymbol{C}^T\boldsymbol{V} \tag{2.38}$$

$$\frac{\partial \boldsymbol{z}}{\partial \boldsymbol{q}} = -\boldsymbol{D}^{-1}\boldsymbol{C}^T\boldsymbol{W} \tag{2.39}$$

Rather, the derivatives $\partial \boldsymbol{x}/\partial \boldsymbol{q}$, $\partial \boldsymbol{y}/\partial \boldsymbol{q}$, $\partial \boldsymbol{z}/\partial \boldsymbol{q}$, and $\partial \boldsymbol{g}/\partial \boldsymbol{q}$ alter on the added conditions set by Eq. (2.25) and, hence, on the assumed other conditions. Different forms of these derivatives have been done by Schek (1973) to set up constraints on the distances between the end nodes, or on the forces acting on the elements or of the cutting lengths (Quagliaroli, 2011; Quagliaroli and Malerba, 2013; Aboul-Nasr and Mourad, 2015).

2.3.2. The Extended Force Density Method

As already shown, the non-linear force density method allows to cope with constraints regarding set relative distances between the nodes, the tensile intensity in the elements and their initial length. A donation which extends the capabilities of the method consists in setting conditions regarding given fixed nodal reactions or, in other words, to establish the positions of a particular number of nodes and, together, to impose the intensity of the reaction force.

Equation (2.1) set the equilibrium equations of a free node of the net. The equilibrium of a fixed node configured in an analogous way, by replacing the forces $\boldsymbol{F}_i$with the end reactions $\boldsymbol{R}_i$, projected in their three components. Via this replacement the equilibrium equations of the fixed nodes are:

$$\begin{bmatrix} \boldsymbol{C}_f^T \boldsymbol{U} \boldsymbol{L}^{-1} \\ \boldsymbol{C}_f^T \boldsymbol{V} \boldsymbol{L}^{-1} \\ \boldsymbol{C}_f^T \boldsymbol{W} \boldsymbol{L}^{-1} \end{bmatrix} \boldsymbol{t} = \begin{bmatrix} \boldsymbol{R}_x \\ \boldsymbol{R}_y \\ \boldsymbol{R}_z \end{bmatrix} \rightarrow \boldsymbol{A}_f \boldsymbol{t} = \boldsymbol{R} \tag{2.40}$$

where

$\boldsymbol{R}_x$, $\boldsymbol{R}_y$, $\boldsymbol{R}_z$ – end reactions

$\boldsymbol{A}_f$ – equilibrium matrix of fixed nodes

Through Eq. (2.40), which derives the end reaction, new form finding conditions can be established. The previous conditions were working on sets of r elements. The constraints on the end reactions act on sets of the n_f fixed nodes. Restraints are placed on a number $s \leq n_f$ of the fixed nodes. Each reaction has three factors. Reactions in each direction are considered distinctly and computation of the variation between the basic value of the reaction $\boldsymbol{R}_i$ is given by Eq. (2.40) and the value of the reactions is imposed by $\boldsymbol{R}_{iv}$.

By writing the equations in matrix form, variations are:

$$\boldsymbol{g}_x = \overline{\boldsymbol{R}}_x - \overline{\boldsymbol{R}}_{xv} = \boldsymbol{0} \tag{2.41}$$

$$\boldsymbol{g}_y = \overline{\boldsymbol{R}}_y - \overline{\boldsymbol{R}}_{yv} = \boldsymbol{0} \tag{2.42}$$

$$\boldsymbol{g}_z = \overline{\boldsymbol{R}}_z - \overline{\boldsymbol{R}}_{zv} = \boldsymbol{0} \tag{2.43}$$

where

$\overline{\boldsymbol{R}}_{(x,y,z)}$ – the values of end reactions in the three directions x, y, z with dimensions $[s \times 1]$

$\overline{\boldsymbol{R}}_{(x,y,z)v}$ – the prescribed values to be imposed in the three directions x, y, z with dimensions $[s \times 1]$

Basic values of reactions in the three directions x, y, z obtained by partitioning the vectors $\overline{\boldsymbol{R}}_{(x,y,z)}$ are as follows:

$$\overline{\boldsymbol{R}}_x = \overline{\boldsymbol{C}}_f^T \boldsymbol{U} \boldsymbol{L}^{-1} \boldsymbol{t} \tag{2.44}$$

$$\overline{\boldsymbol{R}}_y = \overline{\boldsymbol{C}}_f^T \boldsymbol{V} \boldsymbol{L}^{-1} \boldsymbol{t} \tag{2.45}$$

$$\overline{\boldsymbol{R}}_z = \overline{\boldsymbol{C}}_f^T \boldsymbol{W} \boldsymbol{L}^{-1} \boldsymbol{t} \tag{2.46}$$

where

$\overline{\boldsymbol{C}}_f^T$ – transpose of the constrained nodes connectivity matrix with dimensions $[s \times m]$

Matrix $\overline{\boldsymbol{C}}_f^T$ derives from matrix $\boldsymbol{C}_f^T$ by extracting the row corresponding to the nodes to be constrained. It must be pointed out that, working on the nodes, and not on the elements, all the elements and all the terms of the matrices $\boldsymbol{U}$, $\boldsymbol{V}$, $\boldsymbol{W}$, $\boldsymbol{L}^{-1}$and of the vector $\boldsymbol{t}$ are involved in the computation.

2.3.2.1. Jacobian Matrix

Concidering Eq. (2.36), the derivatives of the nodal coordinates on the force densities $\partial\boldsymbol{x}/\partial\boldsymbol{q}$, $\partial\boldsymbol{y}/\partial\boldsymbol{q}$, $\partial\boldsymbol{z}/\partial\boldsymbol{q}$ should be computed as before in Eq. (2.37), (2.38), (2.39) while $\partial\boldsymbol{g}/\partial\boldsymbol{x}$, $\partial\boldsymbol{g}/\partial\boldsymbol{y}$, $\partial\boldsymbol{g}/\partial\boldsymbol{z}$ and $\partial\boldsymbol{g}/\partial\boldsymbol{q}$, depend on the new conditions to be imposed. Vector $\boldsymbol{g}_x$ is considered. The vectors $\boldsymbol{g}_y$ and $\boldsymbol{g}_z$ should be considered in an analogous way. Being $\overline{\boldsymbol{R}}_{xv}$ a constant vector, equality beholds:

$$\frac{\partial\boldsymbol{g}_x}{\partial\boldsymbol{x}} = \frac{\partial\overline{\boldsymbol{R}}_x}{\partial\boldsymbol{x}} \tag{2.47}$$

The dimensions of $\overline{\boldsymbol{R}}_x$, $\boldsymbol{x}$ and $\partial\boldsymbol{g}_x/\partial\boldsymbol{x}$ are $[s \times 1]$, $[n \times 1]$ and $[s \times n]$, respectively.
By deriving Eq. (2.47), it becomes:

$$\frac{\partial\overline{\boldsymbol{R}}_x}{\partial\boldsymbol{x}} = \frac{\partial}{\partial\boldsymbol{x}}\left(\overline{\boldsymbol{C}}_f^T\boldsymbol{U}\boldsymbol{L}^{-1}\boldsymbol{t}\right) = \overline{\boldsymbol{C}}_f^T\frac{\partial}{\partial\boldsymbol{x}}(\boldsymbol{U}\boldsymbol{L}^{-1}\boldsymbol{t}) \tag{2.48}$$

In which, both $\boldsymbol{U}$ and $\boldsymbol{L}^{-1}$ subjected to $\boldsymbol{x}$. By introducing the equation $\boldsymbol{L}^{-1}\boldsymbol{t} = \boldsymbol{q}$ into Eq. (2.48), it transforms to:

$$\frac{\partial\overline{\boldsymbol{R}}_x}{\partial\boldsymbol{x}} = \overline{\boldsymbol{C}}_f^T\frac{\partial}{\partial\boldsymbol{x}}(\boldsymbol{U}\boldsymbol{q}) = \overline{\boldsymbol{C}}_f^T\frac{\partial}{\partial\boldsymbol{x}}(\boldsymbol{Q}\boldsymbol{u}) = \overline{\boldsymbol{C}}_f^T\boldsymbol{Q}\frac{\partial\boldsymbol{u}}{\partial\boldsymbol{x}} \tag{2.49}$$

By using equality $\partial\boldsymbol{u}/\partial\boldsymbol{x} = \boldsymbol{C}$ from Eq. (2.5), Eq. (2.47) becomes:

$$\frac{\partial\boldsymbol{g}_x}{\partial\boldsymbol{x}} = \overline{\boldsymbol{C}}_f^T\boldsymbol{Q}\boldsymbol{C} \tag{2.50}$$

Being $\boldsymbol{u} = \boldsymbol{u}(\boldsymbol{x})$, the derivatives $\partial\boldsymbol{g}_x/\partial\boldsymbol{y}$ and $\partial\boldsymbol{g}_x/\partial\boldsymbol{z}$ are null, as can be seen in the following:

$$\frac{\partial\boldsymbol{g}_x}{\partial\boldsymbol{y}} = \frac{\partial\overline{\boldsymbol{R}}_x}{\partial\boldsymbol{y}} = \overline{\boldsymbol{C}}_f^T\boldsymbol{Q}\frac{\partial\boldsymbol{u}}{\partial\boldsymbol{y}} \equiv \boldsymbol{0} \tag{2.51}$$

$$\frac{\partial\boldsymbol{g}_x}{\partial\boldsymbol{z}} = \frac{\partial\overline{\boldsymbol{R}}_x}{\partial\boldsymbol{z}} = \overline{\boldsymbol{C}}_f^T\boldsymbol{Q}\frac{\partial\boldsymbol{u}}{\partial\boldsymbol{z}} \equiv \boldsymbol{0} \tag{2.52}$$

The equation giving $\partial\boldsymbol{g}_x/\partial\boldsymbol{q}$ is obtained as follows:

$$\frac{\partial\boldsymbol{g}_x}{\partial\boldsymbol{q}} = \frac{\partial\overline{\boldsymbol{R}}_x}{\partial\boldsymbol{q}} = \overline{\boldsymbol{C}}_f^T\frac{\partial}{\partial\boldsymbol{q}}(\boldsymbol{U}\boldsymbol{q}) = \overline{\boldsymbol{C}}_f^T\boldsymbol{U}\frac{\partial\boldsymbol{q}}{\partial\boldsymbol{q}} = \overline{\boldsymbol{C}}_f^T\boldsymbol{U} \tag{2.53}$$

The Jacobian matrix has dimensions $[s \times m]$. The same is made in the other directions. The three Jacobian matrices are finally set up:

$$\boldsymbol{G}_{Rx}^{T} = \overline{\boldsymbol{C}}_{f}^{T}\boldsymbol{U} - \overline{\boldsymbol{C}}_{f}^{T}\boldsymbol{Q}\boldsymbol{C}\boldsymbol{D}^{-1}\boldsymbol{C}^{T}\boldsymbol{U} \tag{2.54}$$

$$\boldsymbol{G}_{Ry}^{T} = \overline{\boldsymbol{C}}_{f}^{T}\boldsymbol{V} - \overline{\boldsymbol{C}}_{f}^{T}\boldsymbol{Q}\boldsymbol{C}\boldsymbol{D}^{-1}\boldsymbol{C}^{T}\boldsymbol{V} \tag{2.55}$$

$$\boldsymbol{G}_{Rz}^{T} = \overline{\boldsymbol{C}}_{f}^{T}\boldsymbol{W} - \overline{\boldsymbol{C}}_{f}^{T}\boldsymbol{Q}\boldsymbol{C}\boldsymbol{D}^{-1}\boldsymbol{C}^{T}\boldsymbol{W} \tag{2.56}$$

With these equations, the problem of finding the geometry of a tensegrity structure for which, in certain fixed nodes, the end reactions assumes prescribed values in the three directions of the reference system, can be solved.

2.3.2.2. Multiple Constraints

It is supposed to assign end reaction forces with arbitrary intensities and directions. This involves a generalization of the method, with the setting of multiple conditions. Let $n_{v,x}$ and $n_{v,z}$ the number of the constrained nodes respectively in x and z directions. By working with the Newton method, at each step the vector $\Delta\boldsymbol{q}$ must satisfy both the conditions on x and z, which are given by:

$$\begin{cases} \boldsymbol{G}_{Rx}^{T}\Delta\boldsymbol{q} = \boldsymbol{r}_{x} = -\boldsymbol{g}_{x} \\ \boldsymbol{G}_{Rz}^{T}\Delta\boldsymbol{q} = \boldsymbol{r}_{z} = -\boldsymbol{g}_{z} \end{cases} \tag{2.57}$$

Alternatively, in matrix form, by:

$$\begin{bmatrix} \boldsymbol{G}_{Rx}^{T} \\ \boldsymbol{G}_{Rz}^{T} \end{bmatrix} \Delta\boldsymbol{q} = \begin{bmatrix} \boldsymbol{r}_{x} \\ \boldsymbol{r}_{z} \end{bmatrix} \tag{2.58}$$

By letting:

$$\boldsymbol{G}_{R}^{T} = \begin{bmatrix} \boldsymbol{G}_{Rx}^{T} \\ \boldsymbol{G}_{Rz}^{T} \end{bmatrix} \tag{2.59}$$

$$\boldsymbol{r}_{xz} = \begin{bmatrix} \boldsymbol{r}_{x} \\ \boldsymbol{r}_{z} \end{bmatrix} \tag{2.60}$$

Equation (2.58) becomes:

$$\boldsymbol{G}_{R}^{T}\Delta\boldsymbol{q} = \boldsymbol{r}_{xz} \tag{2.61}$$

Eq. (2.61) is analogous to Eq. (2.30). Only the dimensions of vectors and matrices change: now the matrix $\boldsymbol{G}_{R}^{T}$ and the vector $\boldsymbol{r}_{xz}$ have dimensions $\left[\left(n_{v,x} + n_{v,z}\right) \times m\right]$ and $\left[\left(n_{v,x} + n_{v,z}\right) \times 1\right]$, respectively, while $\Delta\boldsymbol{q}$ maintains the dimension $[m \times 1]$.

2.4. Form-Finding Process

The proposed form-finding procedure only needs to know the topology of structure regarding the connectivity matrix $\boldsymbol{C}_{s}$, loading conditions and type of each member, i.e. either cable or strut which is under tension or compression, respectively. Based on element type, the initial force density coefficients of cables (tension) are automatically assigned as $+1$ while those of struts (compression) as -1, respectively, as follows:

$$\boldsymbol{q}^0 = \left[\underbrace{+1 \;\; +1 \ldots +1}_{cables} \quad \underbrace{-1 \;\; -1 \ldots -1}_{struts} \right]^T \tag{2.62}$$

Subsequently, the force density matrices $\boldsymbol{D}$ and $\boldsymbol{D_f}$ are calculated from $\boldsymbol{q}^0$ by Eq. (2.19) and (2.20) respectively. After that, the nodal coordinates in x, y, z directions are found from the computation of the Eq. (2.21), (2.22) and (2.23). These nodal coordinates are substituted into Eq. (2.5), (2.6) and (2.7) to define end reaction vectors $\overline{\boldsymbol{R}}_{(x,y,z)}$ through Eq. (2.44), (2.45) and (2.46). Then the difference between the basic value of the reactions $\overline{\boldsymbol{R}}_{(x,y,z)}$ and the value of the reactions that we want to impose $\overline{\boldsymbol{R}}_{(x,y,z)v}$ are calculated (Eq. (2.41), (2.42) and (2.43)). Residual forces $\boldsymbol{r}_{xz}$ are defined by Eq. (2.60). After that, Jacobian matrices are calculated through Eq. (2.54), (2.55) and (2.56) to update the corresponding matrix $\boldsymbol{G}_R^T$ (Eq. (2.59)). Then, by updating the corresponding matrix $\boldsymbol{G}^T$ and vector $\boldsymbol{r}$ and, Eq. (2.33) determines the vector $\Delta\boldsymbol{q}$. The force density vector $\boldsymbol{q}$ is then updated by Eq. (2.34). The iterative process stops when the convergence of residual forces vector $\boldsymbol{r}_{xz}$, with a given small tolerance, is equal to $\boldsymbol{0}$.

Proposed form-finding procedure can simultaneously define two sets of parameters, which are nodal coordinates $[\boldsymbol{x}\ \boldsymbol{y}\ \boldsymbol{z}]$ and force density vector $\boldsymbol{q}$, by applying the following algorithm:

- Step 1: Define $\boldsymbol{C}_s$ by Eq. (2.3) for the given topology of the tensegrity structure.
- Step 2: Specify the type of each member to generate initial force density vector $\boldsymbol{q}^0$ by Eq. (2.62).
- Step 3: Calculate $\boldsymbol{D}$ and $\boldsymbol{D_f}$ using Eq. (2.19) and (2.20), respectively.
- Step 4: Define nodal coordinates $[\boldsymbol{x}\ \boldsymbol{y}\ \boldsymbol{z}]$ through Eq. (2.21), (2.22) and (2.23).
- Step 5: Determine $\overline{\boldsymbol{R}}_{(x,y,z)}$ through Eq. (2.44), (2.45) and (2.46).
- Step 6: Carry out Eq. (2.41), (2.42) and (2.43) to define residual forces vector $\boldsymbol{r}_{xz}$ by Eq. (2.60).
- Step 7: Carry out Eq. (2.54), (2.55) and (2.56) to define the corresponding matrix $\boldsymbol{G}_R^T$ by Eq. (2.59).
- Step 8: Determine vector $\Delta\boldsymbol{q}$ through Eq. (2.33).
- Step 9: Update force density vector $\boldsymbol{q}$ by Eq. (2.34)
- Step 10: The iterative process stops when the convergence of residual forces vector $\boldsymbol{r}_{xz}$, with a given small tolerance, is equal to $\boldsymbol{0}$. The final coordinates and force density vector are the desirable solutions. Otherwise, return to Step 3.
- Step 11: Calculate lengths, forces and preliminary cross-section areas of elements.
- Step 12: Make output file and dxf drawing file.

The procedure of form-finding also outlined in a flowchart presented in Fig. 2.2.

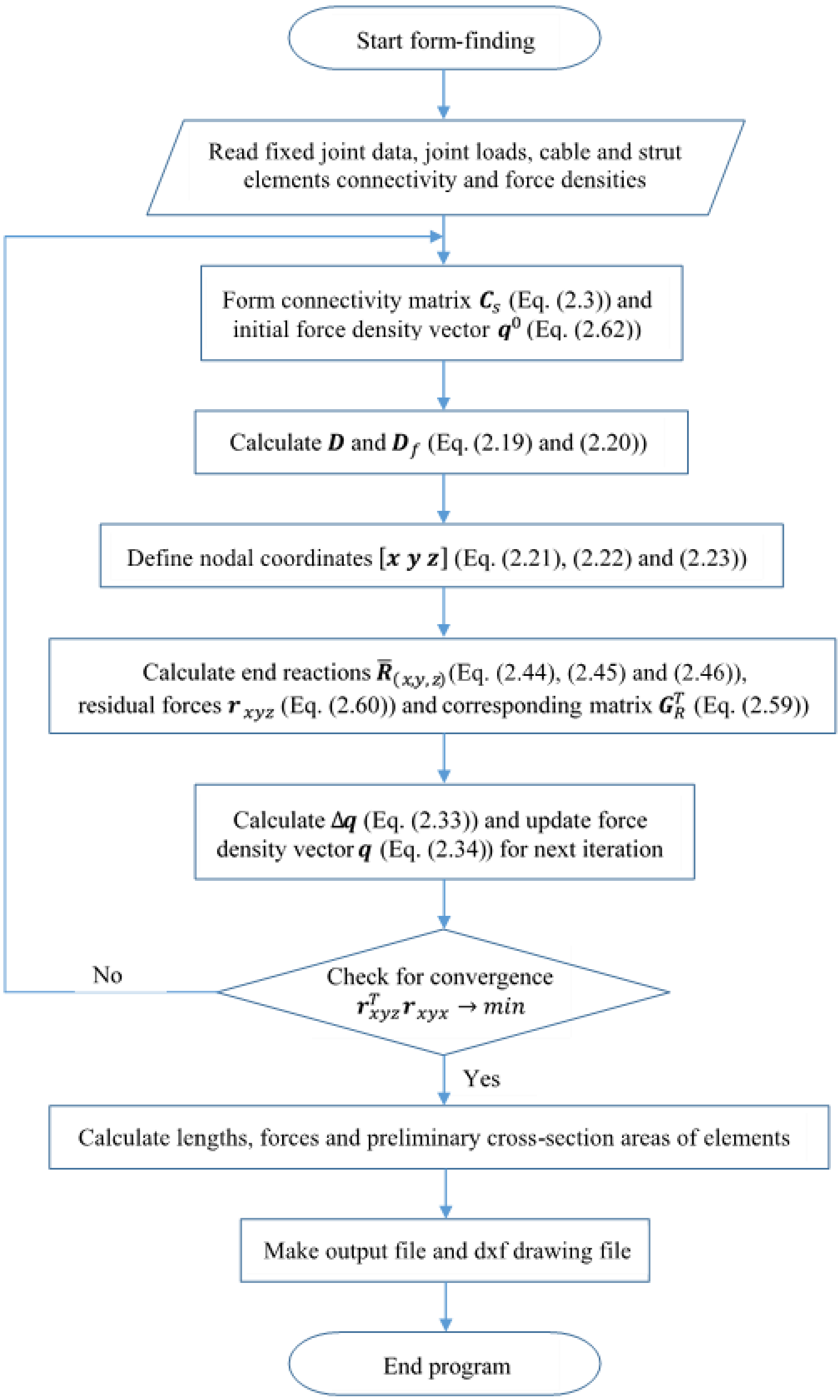

Fig. 2.2. Flowchart of form-finding procedure using EFDM

2.5. Concluding Remarks of Chapter 2

Chapter 2 draws following key findings:

1. Form-finding stands as a self-governing procedure prior to structural analysis and design of flexible structures and defines possible geometry based on an actual set of loads. Geometry becoming a main unknown of the problem makes this procedure uncommon.
2. The overview of existing form-finding methods resulted in FDM to be chosen for future analysis. Due to its flexibility FDM acts on tensegrity structures with no need of assumption referring to geometry or material properties.
3. FDM makes it feasible to find all the possible equilibrium arrangements of a cable strut system with a certainly given connectivity and with given boundary conditions on the nodes.
4. NFDM allows to cope with constraints regarding the set of relative distances between the nodes, pretension of the elements and their initial lengths.
5. To establish the positions of a particular number of nodes and, together, to impose the intensity of the reaction force the EFDM was proposed.
6. The form-finding process through EFDM was introduced and presented. This process lets to determine tensegrity structure form by setting the topology of structure, loading conditions and type of each member, i.e. either cable or strut which is under tension or compression, respectively.

3. TENSEGRITY BRIDGE

In this chapter, a cable-stayed pedestrian bridge made of tensegrity modules is analysed. The form-finding procedure through EFDM described above are done and then analysis of the tensegrity bridge is performed with FEM program. The aim of this work is to determine the particular pretension system (common form-finding problem) under specific loading conditions. Once the initial equilibrium problem has been solved, the structural behavior can be dealt by nonlinear FEA.

In Section 3.1 structural model of tensegrity bridge described. Preliminary data required for the analysis, the form-finding procedure through EFDM, and the non-linear FEA of the bridge performed in Sections 3.2, 3.3 and 3.4, respectively.

3.1. Modelling the Tensegrity Bridge

In this master thesis analysis of the pedestrian cable-stayed bridge composed of tensegrity modules performed. The span of 50.0 m, width of 5.0 m and height of 10.0 m for pylon assumed. The bridge geometry is chosen such that two pedestrians and one cyclist can pass bridge side-by-side. The bridge composed of five tensegrity modules. Each tensegrity module for the bridge is assumed to be 10.0 m long. Rhombic configuration discussed in section 1.5.1 chosen as the main arrangement of cable and strut elements for tensegrity module (see Fig. 3.1).

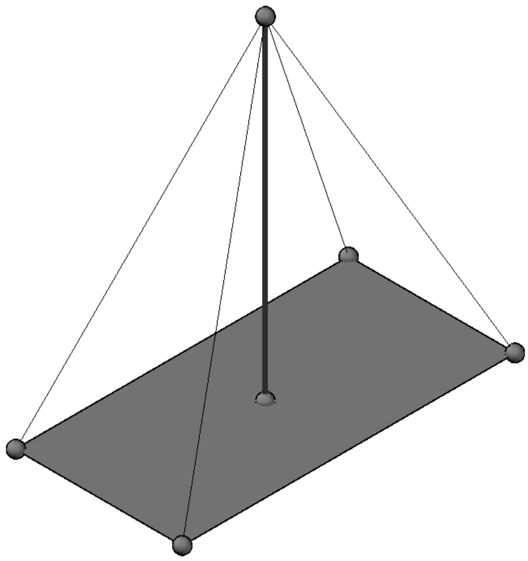

Fig. 3.1. Main module of the tensegrity bridge

The module consists of one strut and four cable elements connected to the deck by pin-joints. To assure the stability of the bridge, nodes by which tensegrity module attached to the deck assumed as fixed in form-finding procedure. Modules one to other connected by nodes at the deck and by cable elements on the top node of the module.

Strut elements in the side modules of the bridge replaced with A-shape pylons (see Fig. 3.2). In form-finding procedure top node of the pylon and end nodes of the deck assumed to be fixed, hence pylon and two side cable elements are not considered in form-finding process and further analysis of the bridge.

The final configuration of tensegrity bridge consists of three modules depicted in Fig. 3.1 and two side modules (see Fig. 3.2). Suggested tensegrity bridge model represented in Fig. 3.3 and 3.4.

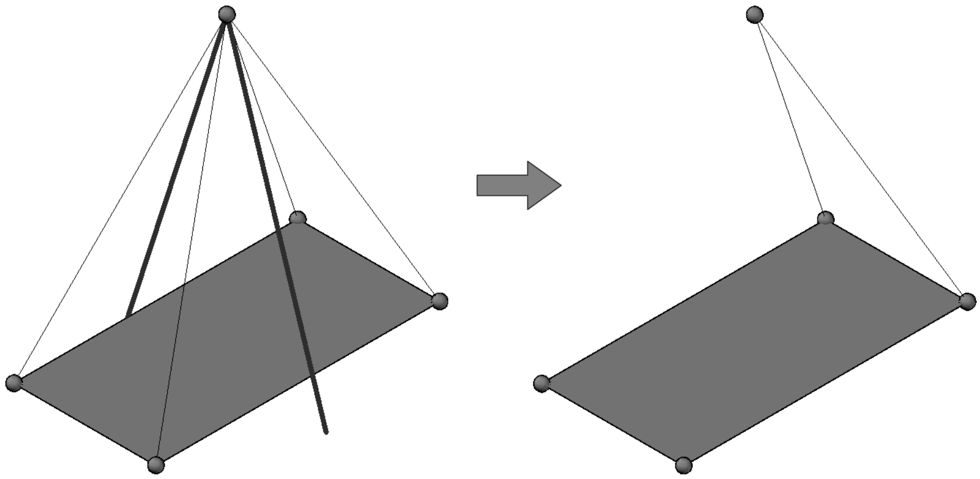

Fig. 3.2. Side module of the tensegrity bridge

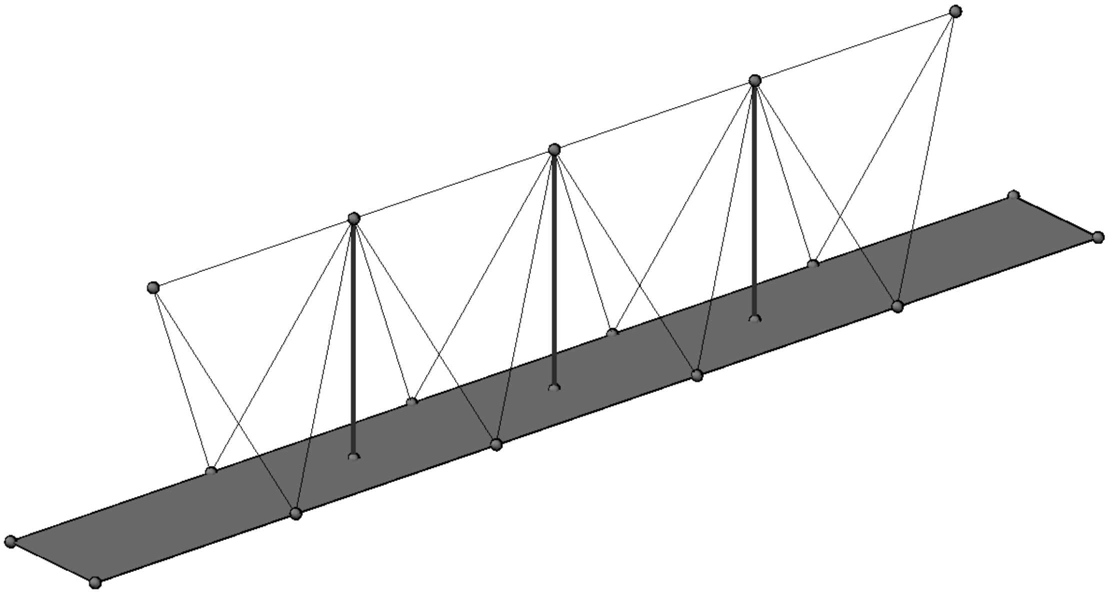

Fig. 3.3. 3D view of computational model of the tensegrity bridge

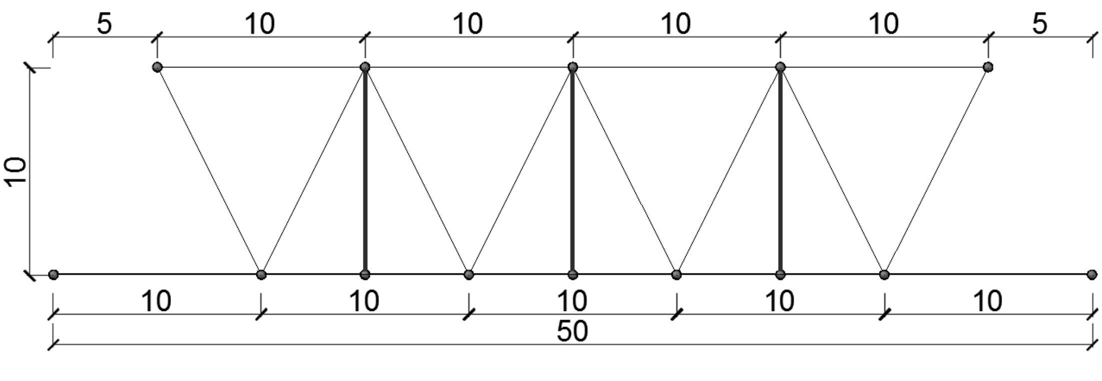

Fig. 3.4. Front view of computational model of the tensegrity bridge

After defining initial configuration of the tensegrity bridge, the form-finding procedure through EFDM and FEA were performed using MATLAB computer program and FEM software SOFiSTiK.

3.2. Preliminary Data Required for Analysis

The dead and live loads considered in this thesis are based on EN 1991-1-1 and EN 1991-2, respectively. The dead load of the bridge contains the structural weight of the bridge. The composite deck made of two longitudinal 0.7 m high side girders (suspended by cable stays) and a slab of uniform thickness of 0.2 m. The slab is supported by the longitudinal girders and the floor beams of 0.5 m high. The floor beams are transverse girders spaced at 5.0 m. Deck construction is only assumed for dead load calculation. The live load of the bridge contains uniformly distributed crowd loading equal to 5.0 kN/m^2.

In Table 3.1 cross-section values for deck members are presented. These values will be used for computation of dead load and later for FEA.

Table 3.1. Cross-section values of deck members for dead load calculation and FEA

Element	Cross-section	Material	Nominal weight, kg/m
Longitudinal girder	HEB 700	Steel	241.0
Transverse girder	HEB 500	Steel	187.0
Slab	5000x200	Concrete	2500.0

Uniformly distributed dead load is:

$$q = \frac{(n_l\gamma_l l_l + n_t\gamma_t l_t + \gamma_s l_s)g}{bL} = 6.3\ kN/m^2 \tag{3.1}$$

where

γ_l, γ_t, γ_s – nominal weights of longitudinal girder, transverse girder and concrete slab, respectively, $[kg/m]$

l_l, l_t, l_l – length of longitudinal girder, transverse girder and concrete slab, respectively, $[m]$

n_l, n_t – number of separate longitudinal girders and transverse girders, respectively

g – gravitational acceleration, $[m/s^2]$

b – width of bridge deck, $[m]$

L – length of the bridge, $[m]$

In this thesis analyses of ULS and SLS are performed in FEA. For form-finding procedure only ULS load combination without pretension forces of cable elements are used. According to EN 1991-2 and EN 1990, the load factors and combinations for pedestrian bridge are as described below.

Load factors and combination for ULS:

$$1.35D + 1.5L + P \tag{3.2}$$

Load factors and combination for SLS:

$$D + L + P \tag{3.3}$$

where

D – the dead load

L – the live load

P – the pretension force of cable elements

Let consider the continuous deck shown in Fig. 3.5. The deck is suspended at the cable stays at the nodes 4-11 and connected to struts at the nodes 12-14. The nodes at the top of the pylons are numbered 15 and 16, and the deck is supported on abutments at the nodes A-D. The geometry of the tensegrity system should be defined through the nodes 1-3. We search for tensegrity system exerting at the internal supports 4-11 a set of forces which equals to the fixed support reactions of the structure shown in Fig. 3.6. In this work, only vertical forces are considered in form-finding process and FEA.

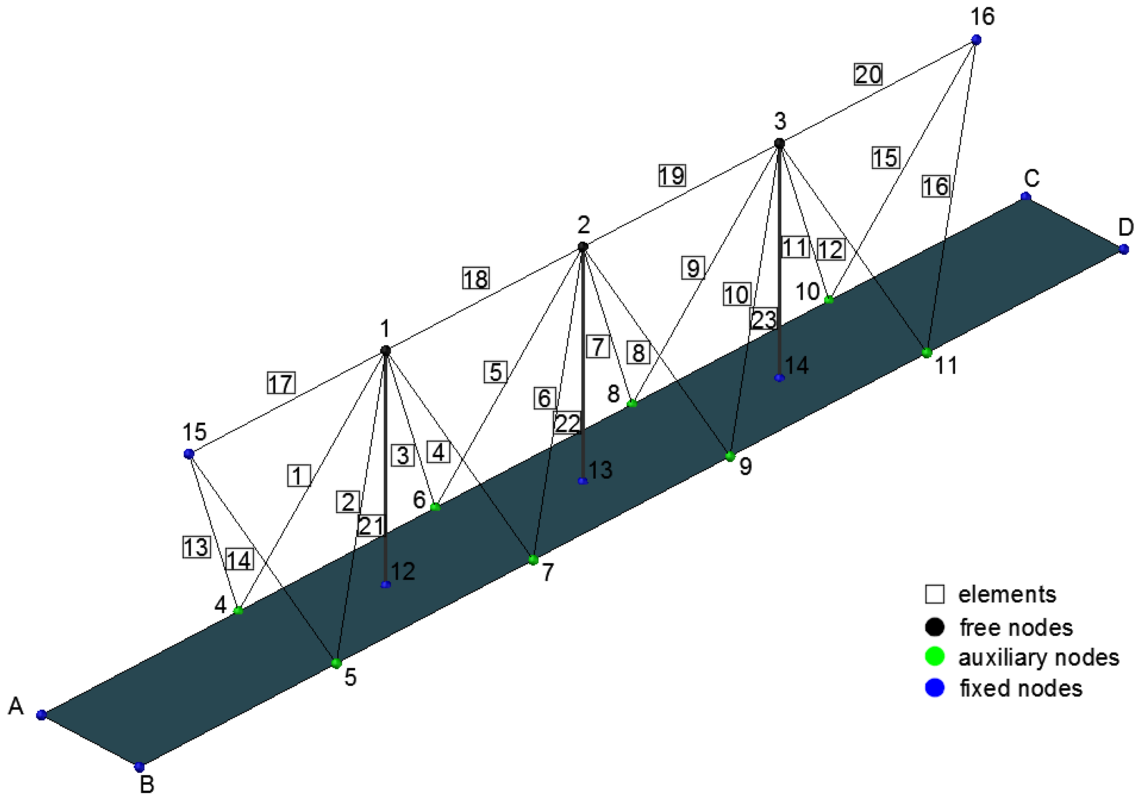

Fig. 3.5. Elements, fixed and free nodes for the tensegrity bridge

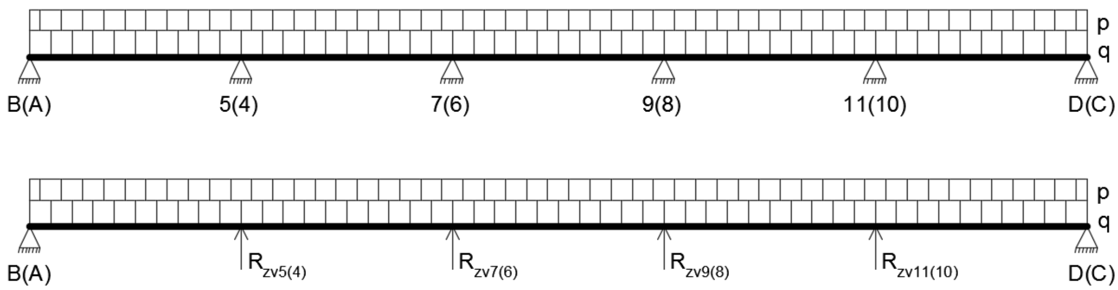

Fig. 3.6. Reaction forces

Reaction force from dead load:

$$R_{z,vd} = \frac{qbl}{2} \tag{3.4}$$

where

$R_{z,vd}$ – reaction force from dead load, $[kN]$
b – width of bridge deck, $[m]$
l – distance between inner supports, $[m]$

Reaction force from live load:

$$R_{z,vl} = \frac{pbl}{2} \tag{3.5}$$

where
$R_{z,vl}$ – reaction force from live load, $[kN]$
b – width of bridge deck, $[m]$
l – distance between inner supports, $[m]$

The characteristic values of the reaction forces to be imposed presented in Table 3.3.

Table 3.2. The characteristic values of the reaction forces

Node group	Nodes	Reaction from dead load $R_{z,vd}, kN$	Reaction from live load $R_{z,vl}, kN$	Total reaction $R_{z,v}, kN$
1	4-11	157.5	125.0	282.5

Material properties used for form-finding process and FEA are presented in the table below (see Table 3.2).

Table 3.3. Material properties of bridge elements

Grade	Yield strength $f_y, N/mm^2$	Proof strength $F_{0.2k}, N/mm^2$	Rope grade $R_r, N/mm^2$	Modulus of elasticity $E, N/mm^2$	Poisons ratio μ
S 355	355.0	-	-	210000.0	0.3
Y1770	-	1570.0	1770.0	195000.0	0.3

S 355 grade steel is used for deck structure and strut elements, and Y1770 grade rope is used for cable stays.

Table 3.4 defines the coordinates of fixed nodes.

Table 3.4. Coordinates of fixed nodes

Node	Coordinate		
	x	y	z
4	10.0	2.5	0.0
5	10.0	-2.5	0.0
6	20.0	2.5	0.0
7	20.0	-2.5	0.0
8	30.0	2.5	0.0
9	30.0	-2.5	0.0
10	40.0	2.5	0.0
11	40.0	-2.5	0.0
12	15.0	0.0	0.0
13	25.0	0.0	0.0
14	35.0	0.0	0.0
15	5.0	1.0	10.0
16	45.0	-1.0	10.0

To generate initial force density vector $\boldsymbol{q}^0$ the type of each member specified and depicted in Fig. 3.4. Thin line represents elements under tension (cables) and thick line represents elements under compression (struts). Table 3.5 defines initial force density vector.

Table 3.5. Initial force density vector

Member	1	2	3	4	5	6	7	8	9	10	11	12
q^0	1	1	1	1	1	1	1	1	1	1	1	1
Member	13	14	15	16	17	18	19	20	21	22	23	
q^0	1	1	1	1	1	1	1	1	-1	-1	-1	

Connectivity matrix is defined in Table 3.6.

Table 3.6. Connectivity matrix of the tensegrity bridge

Member/ node	$\boldsymbol{C}_s$															
	$\boldsymbol{C}$			$\boldsymbol{C}_f$												
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
1	1	0	0	-1	0	0	0	0	0	0	0	0	0	0	0	0
2	1	0	0	0	-1	0	0	0	0	0	0	0	0	0	0	0
3	1	0	0	0	0	-1	0	0	0	0	0	0	0	0	0	0
4	1	0	0	0	0	0	-1	0	0	0	0	0	0	0	0	0
5	0	1	0	0	0	-1	0	0	0	0	0	0	0	0	0	0
6	0	1	0	0	0	0	-1	0	0	0	0	0	0	0	0	0
7	0	1	0	0	0	0	0	-1	0	0	0	0	0	0	0	0
8	0	1	0	0	0	0	0	0	-1	0	0	0	0	0	0	0
9	0	0	1	0	0	0	0	-1	0	0	0	0	0	0	0	0
1	0	0	1	0	0	0	0	0	-1	0	0	0	0	0	0	0
11	0	0	1	0	0	0	0	0	0	-1	0	0	0	0	0	0
12	0	0	1	0	0	0	0	0	0	0	-1	0	0	0	0	0
13	0	0	0	1	0	0	0	0	0	0	0	0	0	0	-1	0
14	0	0	0	0	1	0	0	0	0	0	0	0	0	0	-1	0
15	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	-1
16	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	-1
17	1	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	0
18	1	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
19	0	1	-1	0	0	0	0	0	0	0	0	0	0	0	0	0
20	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	-1
21	1	0	0	0	0	0	0	0	0	0	0	-1	0	0	0	0
22	0	1	0	0	0	0	0	0	0	0	0	0	-1	0	0	0
23	0	0	1	0	0	0	0	0	0	0	0	0	0	-1	0	0

Connectivity matrix of all structure ($\boldsymbol{C}_s$), free nodes ($\boldsymbol{C}$), and fixed nodes ($\boldsymbol{C}_f$) consume dimensions of $[23 \times 16]$, $[23 \times 3]$, and $[23 \times 13]$, respectively. Matrix $\overline{\boldsymbol{C}}_f$ derived from matrix $\boldsymbol{C}_f$ by extracting the columns corresponding to the nodes to be constrained consume dimensions of $[23 \times 8]$.

3.3. Form-Finding of Tensegrity Bridge

Multi-paradigm numerical computing environment MATLAB selected to perform form-finding of above-presented tensegrity bridge. A proprietary programming language developed by MathWorks, MATLAB allows matrix manipulations, plotting of data, and implementation of algorithms.

Form-finding algorithm written with MATLAB outlined in section 3.3.1 and the results of a search of the new form of the bridge presented in section 3.3.2.

3.3.1. Computational Algorithm of Tensegrity Bridge

EFDM algorithm formulated in section 2.4 is composed using MATLAB computer program. Iterative process is performed until the convergence with a small given tolerance reaches 0 value. After that calculation of the elements axial forces and geometrical parameters (length, cross section area of strut and cable elements) are done.

Calculation process of the elements axial forces and geometrical parameters are performed as follows:

- Calculation of length of cable and strut elements (Eq. 3.6)
- Calculation of elements axial forces, from concept of force density $q_{ij} = T_{ij}/L_{ij}$ (Eq. 3.7)
- Estimation of strut cross-section area according to ULS conditions (EN 1993-1-1) (Eq. 3.8);
- Estimation of cable cross-section area according to ULS conditions (EN 1993-1-11) (Eq. 3.9).

Length of the elements:

$$\boldsymbol{l} = diag(\boldsymbol{L}) = diag\left(\sqrt{\boldsymbol{U}^2 + \boldsymbol{V}^2 + \boldsymbol{W}^2}\right) \tag{3.6}$$

Axial forces in elements:

$$\boldsymbol{t} = \boldsymbol{Lq} \tag{3.7}$$

Cross-section area of strut elements:

$$A_s = \sqrt{\frac{F_s \gamma_{M0}}{f_y}} \tag{3.8}$$

where

F_s – axial force in strut elements, $[kN]$

γ_{M0} – the partial safety factor equal to 1.0 (EN 1993-1-1)

f_y – yield strength equal to 355.0 (LST EN 1993-1-1), $[N/mm^2]$

Cross-section area of cable elements:

$$A_c = min\left(\sqrt{\frac{1500 F_c \gamma_R \pi}{4 K R_r k_e}}; \sqrt{\frac{1000 F_c \gamma_R}{F_{0.2k}}}\right) \tag{3.9}$$

where

F_c – axial force in cable elements, $[kN]$

γ_R – the partial safety factor equal to 1.0 (EN 1993-1-11)

K – the minimum breaking force factor taking account of the spinning loss equal to 0.51 (EN 12385-2)

$F_{0.2k}$ – the characteristic value of the proof strength of the tension component equal to 1370.0 (EN 10264), $[N/mm^2]$

R_r – the rope grade equal to 1570.0 (EN 12385-2), $[N/mm^2]$

k_e – the loss factor equal to 1.0 (EN 1993-1-11)

Algorithm written with MATLAB is presented below.

```
clear
clc

% Form finding algorithm of tensegrity bridge
% Input data
% Number of elements
m = 23;
% Number of free nodes
n = 3;
% Number of auxiliary nodes
n_a = 8;
% Number of constrained nodes
n_c = 5;
% Number of fixed nodes
n_f = n_a + n_c;
% Connectivity matrix of free nodes
C = [1,0,0;1,0,0;1,0,0;1,0,0;0,1,0;0,1,0;0,1,0;0,1,0;0,0,1;0,0,1;0,0,1;0,0,1;0,0,0;0,0,0;0,0,0;0,
0,0;1,0,0;1,-1,0;0,1,-1;0,0,1;1,0,0;0,1,0;0,0,1];
% Connectivity matrix of auxiliary nodes
C_a = [-1,0,0,0,0,0,0,0;0,-1,0,0,0,0,0,0;0,0,-1,0,0,0,0,0;0,0,0,-1,0,0,0,0;0,0,-1,0,0,0,0,0;0,0,0,-
1,0,0,0,0;0,0,0,0,-1,0,0,0;0,0,0,0,0,-1,0,0;0,0,0,0,-1,0,0,0;0,0,0,0,0,-1,0,0;0,0,0,0,0,0,-1,0;0,0,
0,0,0,0,0,-1;1,0,0,0,0,0,0,0;0,1,0,0,0,0,0,0;0,0,0,0,0,0,1,0;0,0,0,0,0,0,0,1;0,0,0,0,0,0,0,0;0,0,0,
0,0,0,0,0;0,0,0,0,0,0,0,0;0,0,0,0,0,0,0,0;0,0,0,0,0,0,0,0;0,0,0,0,0,0,0,0;0,0,0,0,0,0,0,0];
% Connectivity matrix of constrained nodes
C_c = [0,0,0,0,0;0,0,0,0,0;0,0,0,0,0;0,0,0,0,0;0,0,0,0,0;0,0,0,0,0;0,0,0,0,0;0,0,0,0,0;0,0,0,0,0;
0,0,0,0,0;0,0,0,0,0;0,0,0,0,0;0,0,0,-1,0;0,0,0,-1,0;0,0,0,0,-1;0,0,0,0,-1;0,0,0,-1,0;0,0,0,0,0;0,
0,0,0,0;0,0,0,0,-1;-1,0,0,0,0;0,-1,0,0,0;0,0,-1,0,0];
% Connectivity matrix of fixed nodes
C_f = [C_a C_c];
% Connectivity matrix of all structure
C_s = [C C_f];
% Coordinates of auxiliary nodes in x, y and z direction
x_a = [10;10;20;20;30;30;40;40];
y_a = [2.5;-2.5;2.5;-2.5;2.5;-2.5;2.5;-2.5];
z_a = [0;0;0;0;0;0;0;0];
% Coordinates of constrained nodes in x, y and z direction
x_c = [15;25;35;5;45];
y_c = [0;0;0;1;-1];
z_c = [0;0;0;10;10];
% Coordinates of fixed nodes in x, y and z direction
x_f = [x_a; x_c];
y_f = [y_a; y_c];
z_f = [z_a; z_c];
% Force density vector
q = [1;1;1;1;1;1;1;1;1;1;1;1;1;1;1;1;1;1;1;1;-1;-1;-1];
% Diagonal matrix of vector q
Q = diag(q);
% Force density matrix of free nodes
```

```
D = C'*Q*C;
% Force density matrix of fixed nodes
D_f = C'*Q*C_f;
% Free nodes loads in x, y and z direction
p_x = [0;0;0];
p_y = [0;0;0];
p_z = [0;0;0];
% New coordinates of free nodes in x, y and z direction
x = D^(-1)*(p_x-D_f*x_f);
y = D^(-1)*(p_y-D_f*y_f);
z = D^(-1)*(p_z-D_f*z_f);
% Coordinate differences in x, y and z direction
u = C*x+C_f*x_f;
v = C*y+C_f*y_f;
w = C*z+C_f*z_f;
% Diagonal matrices of vectors u, v and w
U = diag(u);
V = diag(v);
W = diag(w);
% Value of the calculated reaction in x, y and z direction
R_x = C_a'*U*q;
R_y = C_a'*V*q;
R_z = C_a'*W*q;
% Value of the imposed reaction in x, y and z direction
R_xv = [0;0;0;0;0;0;0;0];
R_yv = [0;0;0;0;0;0;0;0];
R_zv = [-400.125;-400.125;-400.125;-400.125;-400.125;-400.125;-400.125;-400.125];
% The difference between calculated and imposed reactions
g_x = R_x-R_xv;
g_y = R_y-R_yv;
g_z = R_z-R_zv;
% Constraints
r_xz = [-g_x;-g_z];
% Jacobian matrices
G_Rx = C_a'*U-C_a'*Q*C*D^(-1)*C'*U;
G_Ry = C_a'*V-C_a'*Q*C*D^(-1)*C'*V;
G_Rz = C_a'*W-C_a'*Q*C*D^(-1)*C'*W;
% Corresponding matrix
G_R = [G_Rx;G_Rz];
% Change in force density vector
q_delta = G_R'*pinv(G_R*G_R')*r_xz;
% Convergence
k = r_xz'*r_xz;
% Iterations
while k >= 0.001
   q = q + q_delta;
   Q = diag(q);
   D = C'*Q*C;
   D_f = C'*Q*C_f;
   x = D^(-1)*(p_x-D_f*x_f);
   y = D^(-1)*(p_y-D_f*y_f);
   z = D^(-1)*(p_z-D_f*z_f);
   u = C*x+C_f*x_f;
   v = C*y+C_f*y_f;
   w = C*z+C_f*z_f;
```

```
    U = diag(u);
    V = diag(v);
    W = diag(w);
    R_x = C_a'*U*q;
    R_y = C_a'*V*q;
    R_z = C_a'*W*q;
    g_x = R_x-R_xv;
    g_y = R_y-R_yv;
    g_z = R_z-R_zv;
    r_xz = [-g_x;-g_z];
    G_Rx = C_a'*U-C_a'*Q*C*D^(-1)*C'*U;
    G_Ry = C_a'*V-C_a'*Q*C*D^(-1)*C'*V;
    G_Rz = C_a'*W-C_a'*Q*C*D^(-1)*C'*W;
    G_R = [G_Rx;G_Rz];
    q_delta = G_R'*pinv(G_R*G_R')*r_xz;
    k = r_xz'*r_xz;
end
% Elements length
L = sqrt(U^2 + V^2 + W^2);
l = diag(L);
% Axial forces
t = L*q;
% Maximum force in strut elements
F_s = max(abs(t([21:23])));
% Maximum force in cable elements
F_c = max(t([1:20]));
% The partial safety factors
gama_M0 = 1;
gama_R = 1;
% Yield strength
f_y = 355;
% The characteristic value of the proof strength of the tension component
F_02k = 1570;
% The rope grade
R_r = 1770;
% The minimum breaking force factor taking account of the spinning loss
K = 0.51;
% The loss factor
k_e = 1;
% The minimum cross-section area of strut elements
A_s = (F_s*gama_M0)/(f_y*10^-3);
% The minimum cross-section area of cable elements
A_c = min((1500*F_c*gama_R*pi)/(4*K*R_r*k_e),(1000*F_c*gama_R)
/(F_02k));
% Results
disp(x)
disp(y)
disp(z)
disp(q)
disp(l)
disp(t)
disp(F_s)
disp(F_c)
disp(A_s)
disp(A_c)
```

```
scatter3(x_f,y_f,z_f,'*')
hold on
scatter3(x,y,z,'*')
grid on
```

Results of form-finding process and computations of the elements axial forces and geometrical parameters presented in Section 3.2.3.

3.3.2. Results of Form-Finding

The output data of the algorithm compiled with MATLAB presented in tables and figures below. Table 3.7 gives new free node's coordinates.

Table 3.7. New free nodes coordinates

Node	Coordinate		
	x	y	z
1	15.776	0.401	5.703
2	25.000	0.000	4.112
3	34.224	-0.401	5.703

After we get free nodes coordinates final configuration of the tensegrity bridge structure can be defined. 3D view of computational model of the tensegrity bridge depicted in Fig. 3.7 also front, top, and side views of the model shown in Figs. 3.8, 3.9, and 3.10, respectively.

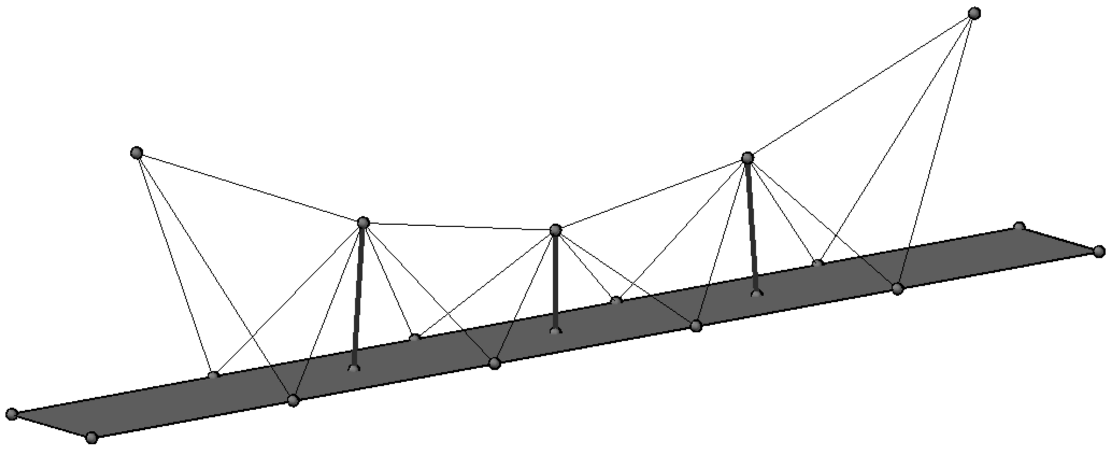

Fig. 3.7. 3D view of the final configuration of the tensegrity bridge

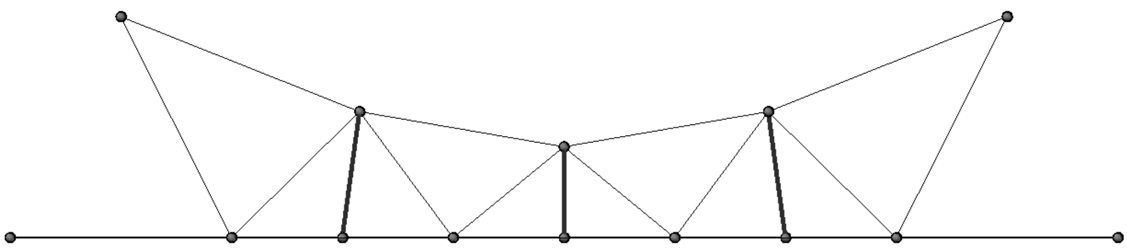

Fig. 3.8. Front view of the final configuration of the tensegrity bridge

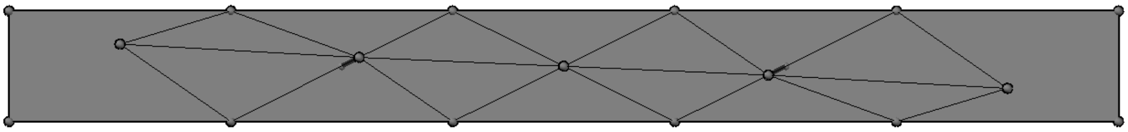

Fig. 3.9. Top view of the final configuration of the tensegrity bridge

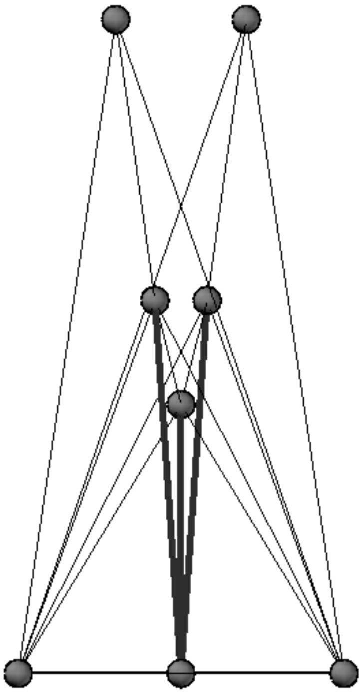

Fig. 3.10. Side view of the final configuration of the tensegrity bridge

Updated force density vector ***q*** is presented in Table 3.8.

Table 3.8. Force density vector

Member	1	2	3	4	5	6
Force density*q*	23.19	23.19	43.59	43.59	36.82	36.82
Member	7	8	9	10	11	12
Force density*q*	36.82	36.82	43.59	43.59	23.19	23.19
Member	13	14	15	16	17	18
Force density*q*	26.79	26.79	26.79	26.79	123.35	126.84
Member	19	20	21	22	23	
Force density*q*	126.84	123.35	-76.01	-49.25	-76.01	

Based on element type, the updated force density coefficients vector of cables (tension) automatically gets positive values while those of struts (compression) gets negative ones.

Calculation results of the elements axial forces and geometrical parameters (length, cross section area of strut and cable elements) presented in Table 3.9.

Table 3.9. Calculation results of the elements axial forces and geometrical parameters

Group	Member	Length l, m	Axial force t, kN	Maximum force F, kN	Minimum cross-section area A, mm^2
Cable elements	1	8.384	194.42	1432.86	1050.0
	2	8.619	199.89		
	3	7.401	322.61		
	4	7.667	334.21		
	5	6.940	255.55		
	6	6.940	255.55		
	7	6.940	255.55		
	8	6.940	255.55		
	9	7.667	334.21		
	10	7.401	322.61		
	11	8.619	199.89		
	12	8.384	194.42		
	13	11.281	302.16		
	14	11.715	313.81		
	15	11.715	313.81		
	16	11.281	302.16		
	17	11.616	1432.86		
	18	9.369	1188.26		
	19	9.369	1188.26		
	20	11.616	1432.86		
Strut elements	21	5.769	-438.68	438.68	1240.0
	22	4.112	-202.64		
	23	5.769	-438.68		

These results were obtained after an initial calculation was performed with a fixed tolerance equal to 0.001 for their respective convergence criteria (Eq. 2.35). All elements lengths and axial forces were defined. According to maximum forces of cable and strut elements minimum required cross-section areas were estimated. After estimation of all parameters described before, FEA of tensegrity bridge can be performed.

3.4. FEA of Tensegrity Bridge

FEA and CAD software SOFiSTiK for civil and structural engineering modeling, analysis, design, and detailing selected to perform FEA of in form-finding process gotten tensegrity bridge structure.

Camputanional bridge scheme describred in section 3.4.1 and the results of FEA of the tensegrity bridge presented in section 3.4.2.

3.4.1. Finite Element Model of Tensegrity Bridge

In this work FEA purpose is to check if the results obtained from form-finding process is reliable and can be used for further analysis of the analysed structure. FEA has been performed in order to ensure that the stability and the bearing capacity of the elements, as well as, deflection of the deck, and natural frequencies conform the requirements of EN 1993-1-1, EN 1993-1-11, and EN 1991-2.

Strut and cable elements bearing capacity was checked by using design tool in FEM software SOFiSTiK. Maximal deflections of the deck should be limited to 1/250 of span length.

Natural frequencies (corresponding to vertical, horizontal, torsional vibrations) of the main structure of the bridge deck should be:

- vertical, with a frequency that can range between 1 and 3 Hz, and
- horizontal, with a frequency that can range between 0,5 and 1,5 Hz.

Groups of joggers may cross a footbridge with a frequency of 3 Hz.

FEA has been done in three main stages:

- Pre-processing, in which a finite element mesh of the geometry obtained in form-finding process is developed and material properties, boundary conditions and loads are applied.
- Solution, during which the program solves for the displacements, strains, and stresses.
- Post-processing, in which the results are obtained.

The 3D model obtained with SOFiSTiK shown in Figs. 3.11, 3.12, 3.13, and 3.14. Boundary conditions are indicated as red highlight.

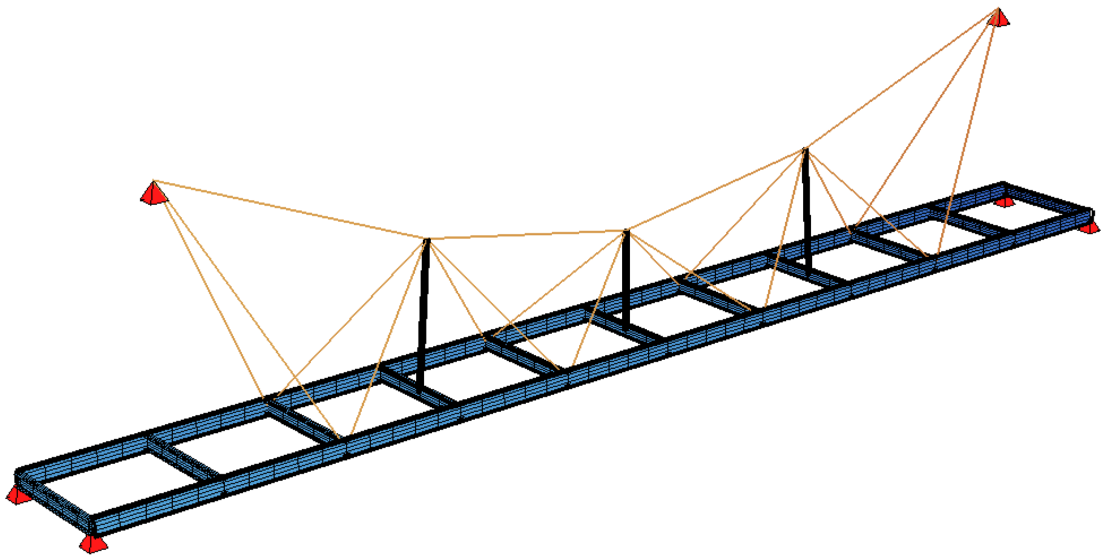

Fig. 3.11. 3D view of computational model of the tensegrity bridge for FEA

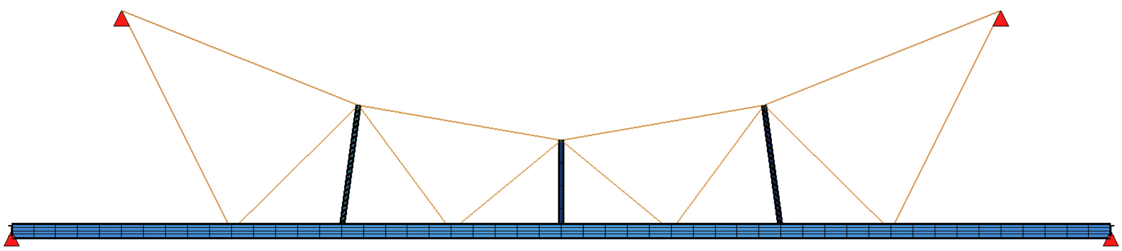

Fig. 3.12. Front view of computational model of the tensegrity bridge for FEA

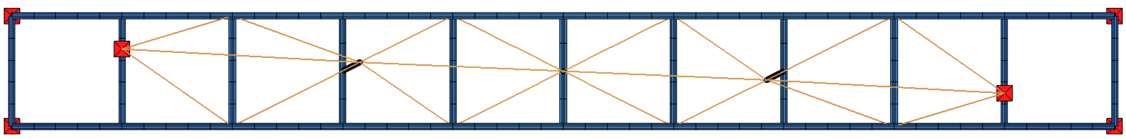

Fig. 3.13. Top view of computational model of the tensegrity bridge for FEA

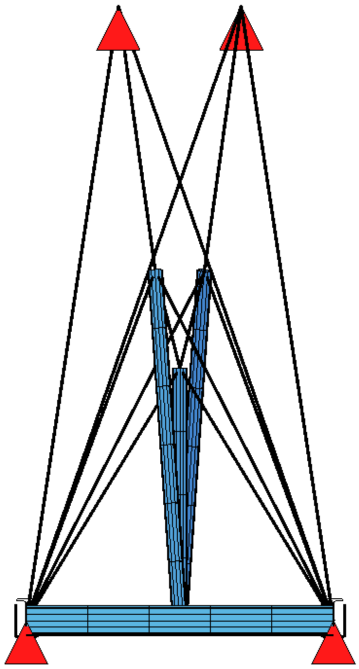

Fig. 3.14. Side view of computational model of the tensegrity bridge for FEA

Cable and strut elements chosen according to minimum cross-section area gotten during form-finding procedure described in Table 3.10.

Table 3.10. Cross-section values of cable and strut members for FEA

Element	Cross-section	Material	Nominal weight, kg/m
Cable stays	D60 1x127	Steel	10.0
Strut	TUBE 244.5x8	Steel	46.7

Material properties used for the tensegrity bridge modelling with FEA software SOFiSTiK are described in section 3.2.

Two load combinations have been used to the 3D model of the tensegrity bridge. One is to check for ULS and second for SLS. Dead and live loads and loads combinations are described in Section 3.2. Pretension forces of cable elements obtained from form-finding procedure are presented in Table 3.11. Only symmetrical loading is considered in FEA. Loaded tensegrity bridge structure depicted in Figs. 3.15, 3.16, and 3.17.

Table 3.11. Pretension forces in cable elements

Member	1	2	3	4	5
Post-tensioning force P, kN	194.42	199.89	322.61	334.21	255.55
Member	6	7	8	9	10
Post-tensioning force P, kN	255.55	255.55	255.55	334.21	322.61
Member	11	12	13	14	15
Post-tensioning force P, kN	199.89	194.42	302.16	313.81	313.81
Member	16	17	18	19	20
Post-tensioning force P, kN	302.16	1432.86	1188.26	1188.26	1432.86

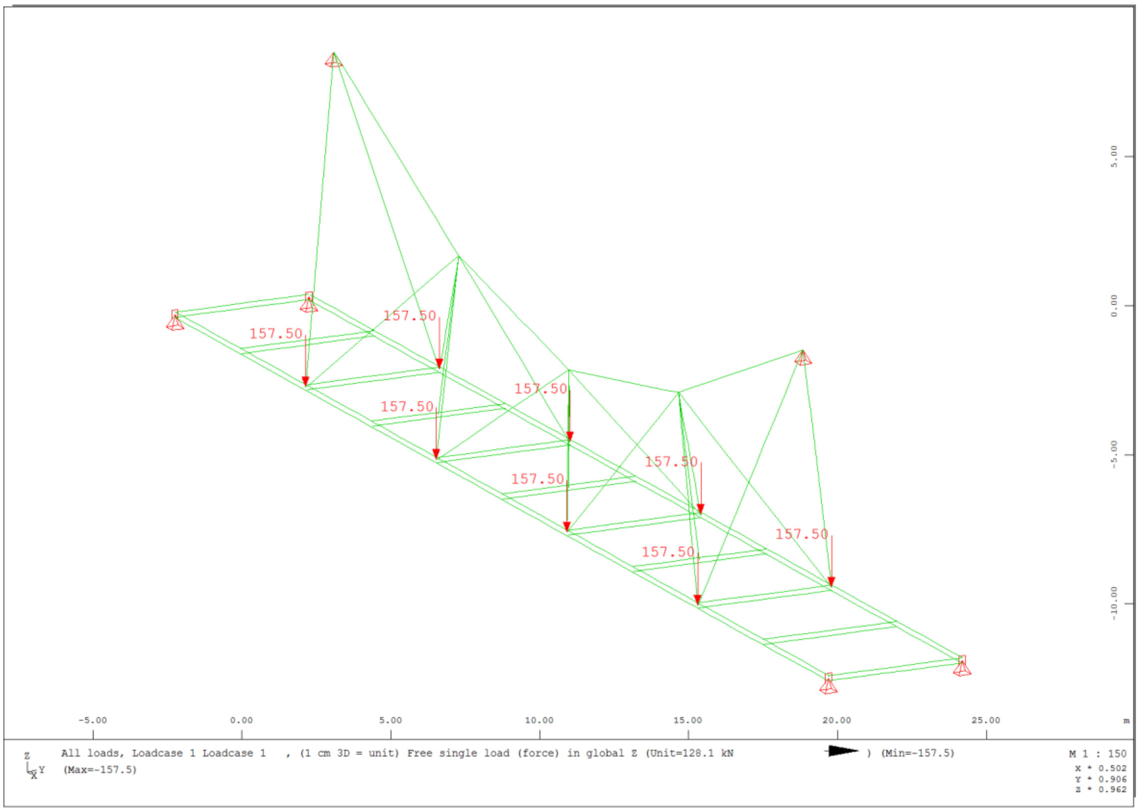

Fig. 3.15. Dead load

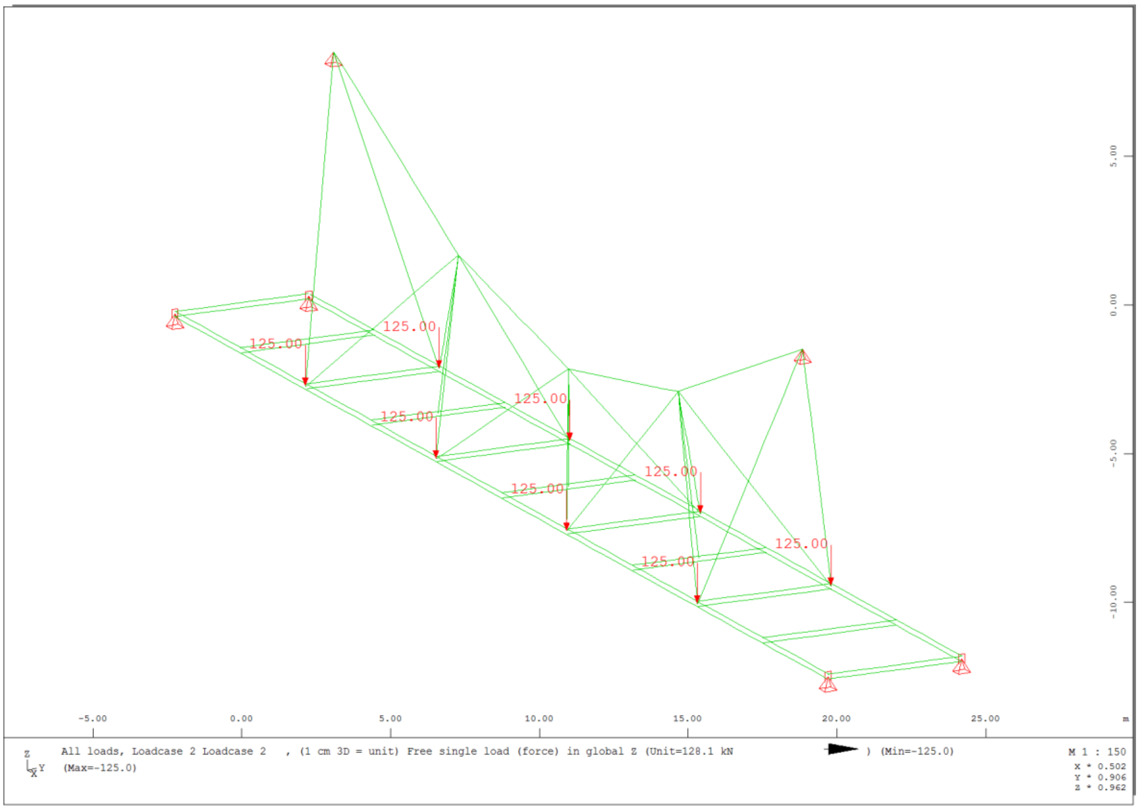

Fig. 3.16. Live load

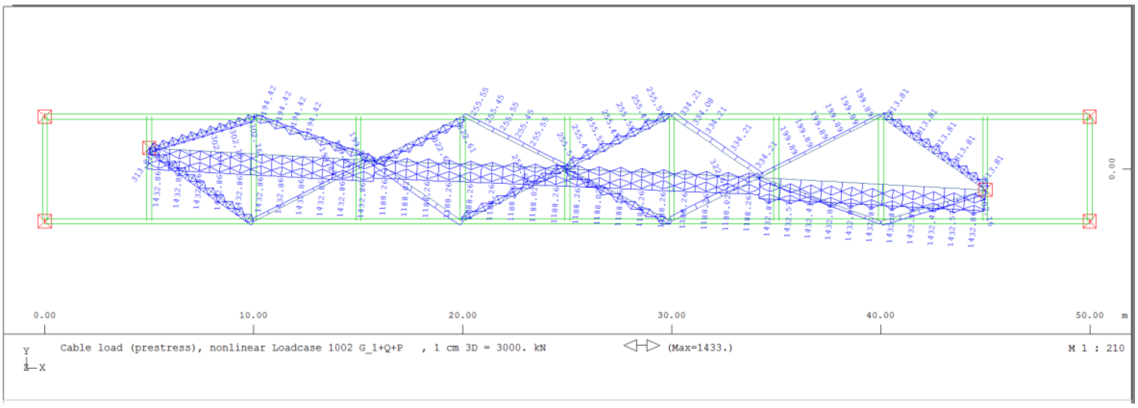

Fig. 3.17. Pretension forces

After first stage of FEA was performed and all needed input data were developed and applied in SOFiSTiK software second stage of FEA was initiated. Results of FEA represented in next section.

3.4.2. Results of FEA

FEA result are presented in figures and tables below. Bending moments diagram presented in Fig. 3.18, while axial forces diagrams of cable and strut elements depicted in Figs. 3.19 and 3.20, respectively.

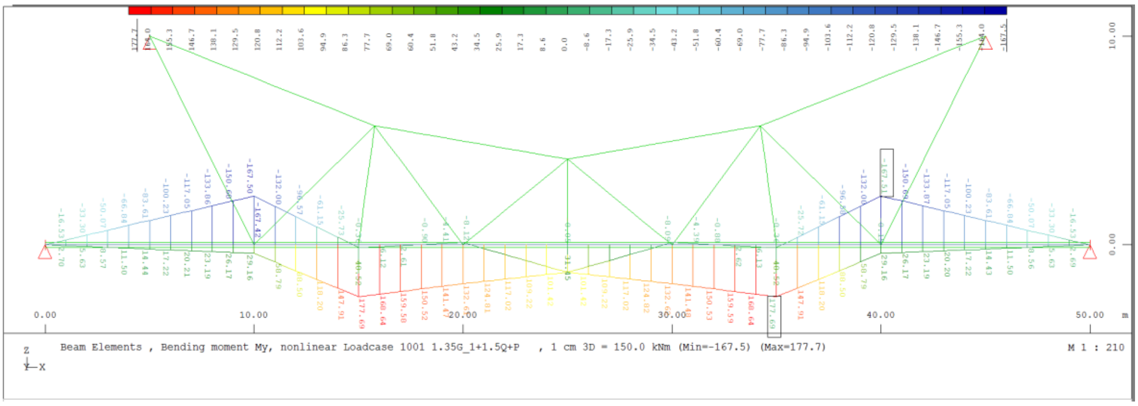

Fig. 3.18. Diagram of bending moments of deck structure

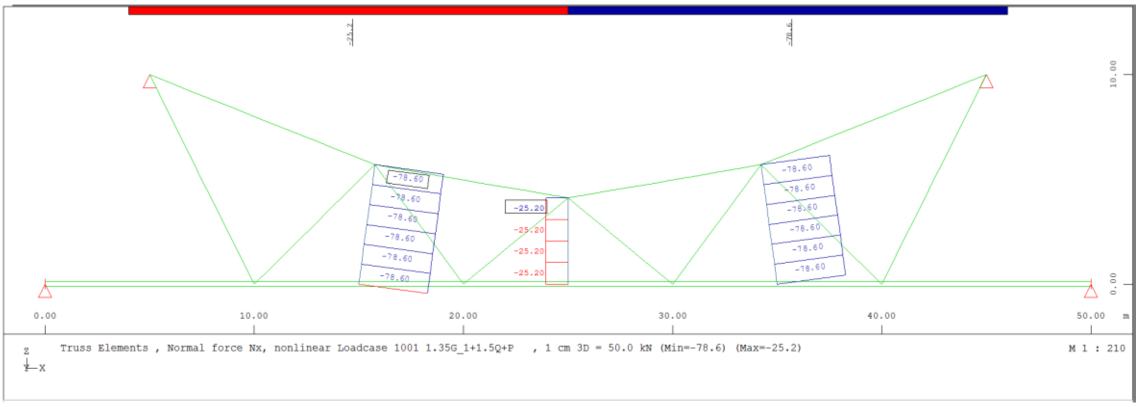

Fig. 3.19. Diagram of axial forces of strut elements

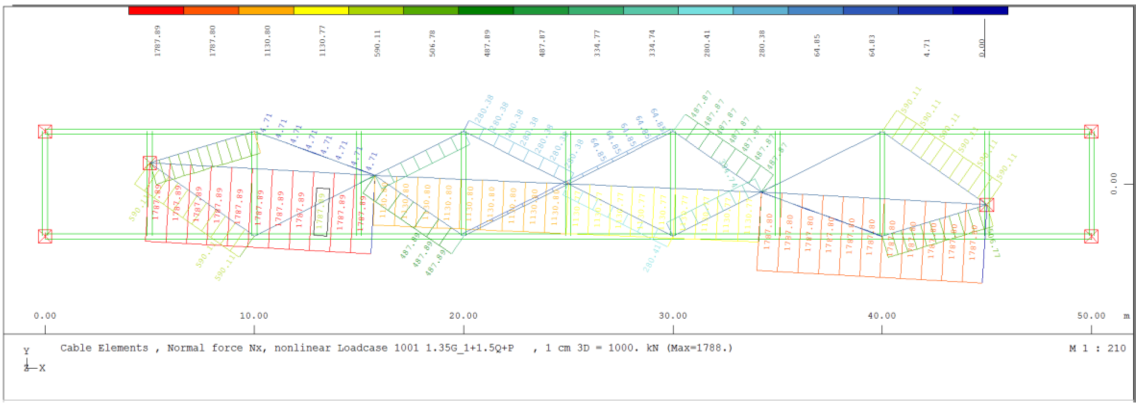

Fig. 3.20. Diagram of axial forces of cable elements

As can be seen in Fig. 3.18 maximum and minimum moments from ULS load combination in deck structure are $M_{max} = 177.7\ kN/m$ and $M_{min} = -167.5\ kN/m$, respectively. Deference between them is 6.1%, that shows good distribution of bending moments in deck structure.

Axial forces acting in cable and strut elements and their bearing capacity calculation results are summarized in Table 3.12.

Table 3.12. Bearing capacity of cable and strut elements

Group	Member	Axial force t, kN	Maximum force N_{Ed}, kN	Bearing capacity N_{Rd}, kN	Buckling resistance N_{Rd}, kN
Cable elements	1	4.71	1787.80	2166.48	-
	2	0.00			
	3	334.77			
	4	487.89			
	5	280.38			
	6	64.83			
	7	64.83			
	8	280.41			
	9	487.87			
	10	334.74			
	11	0.00			
	12	4.73			
	13	506.78			
	14	590.11			
	15	590.11			
	16	506.77			
	17	1787.89			
	18	1130.80			
	19	1130.77			
	20	1787.80			
Strut elements	21	-78.60	78.60	2110.12	524.36
	22	-25.20			
	23	-78.60			

According to the results of the analysis, bearing capacity of the elements is sufficient.

After the analysis of ULS conditions, analysis of the SLS conditions were performed. Deck deflection and natural frequencies of the deck structure were calculated.

Calculation result of deck deflection presented in Fig. 3.21.

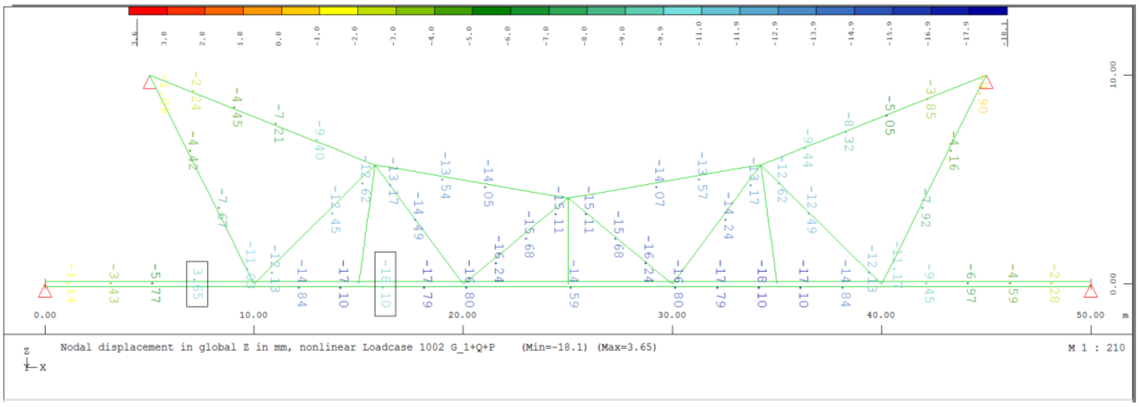

Fig. 3.21. Deflection of the tensegrity bridge deck structure

In this case maximal deck deflections should be limited to 200.0 mm. As can be seen in Fig. 3.21 maximum deflection from SLS load combination in deck structure is equal to 18.1 mm. Condition of deck deflection is satisfied.

Eight forms of natural frequencies of the deck structure were calculated. Calculation results presented in Figs. 3.22 to 3.29 and Table 3.13.

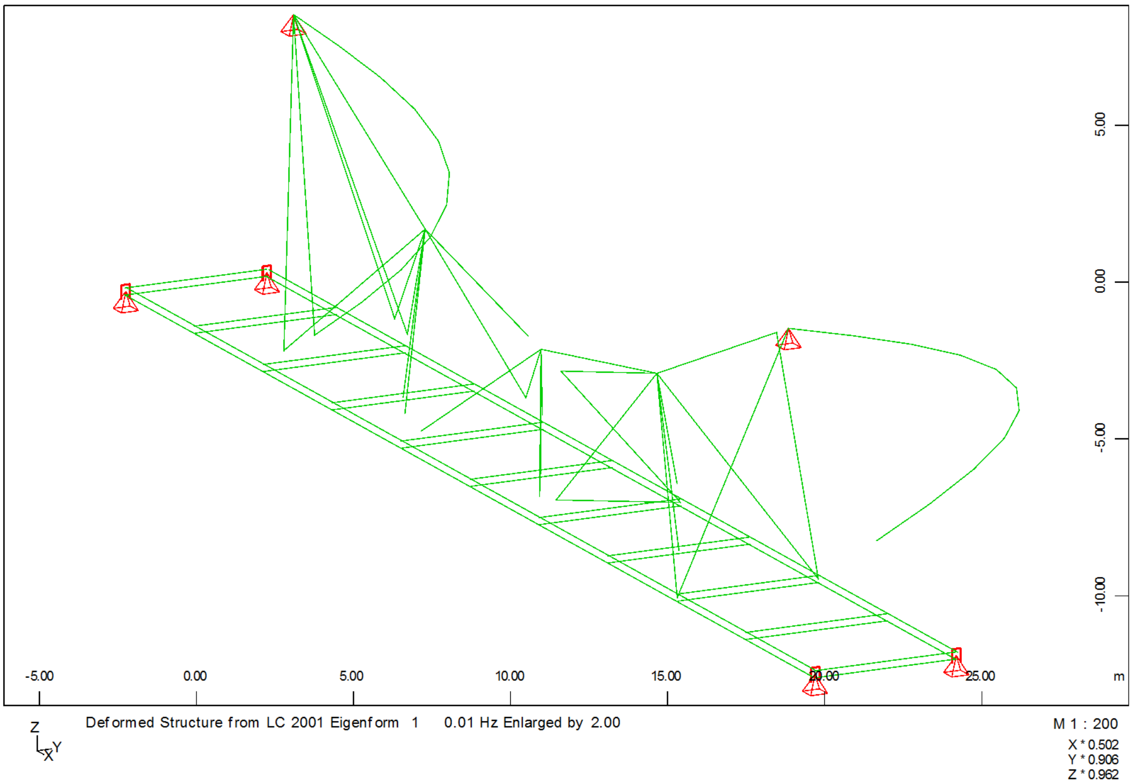

Fig. 3.22. 1st eigenform

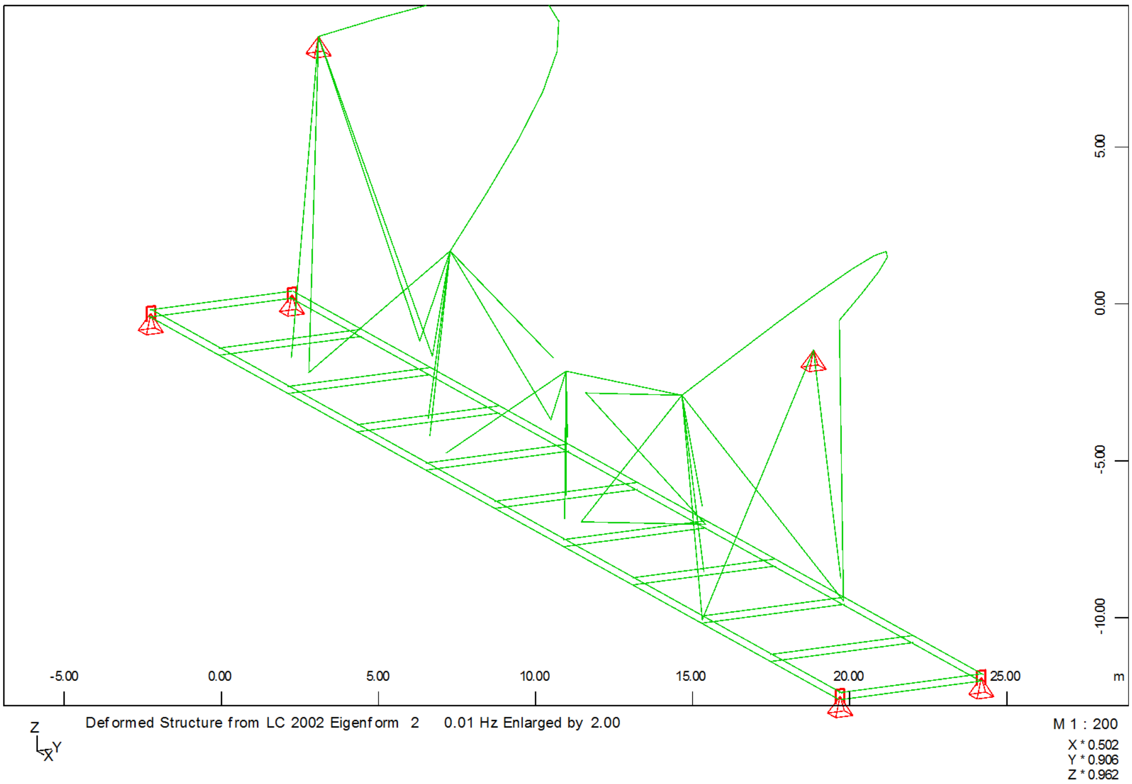

Fig. 3.23. 2nd eigenform

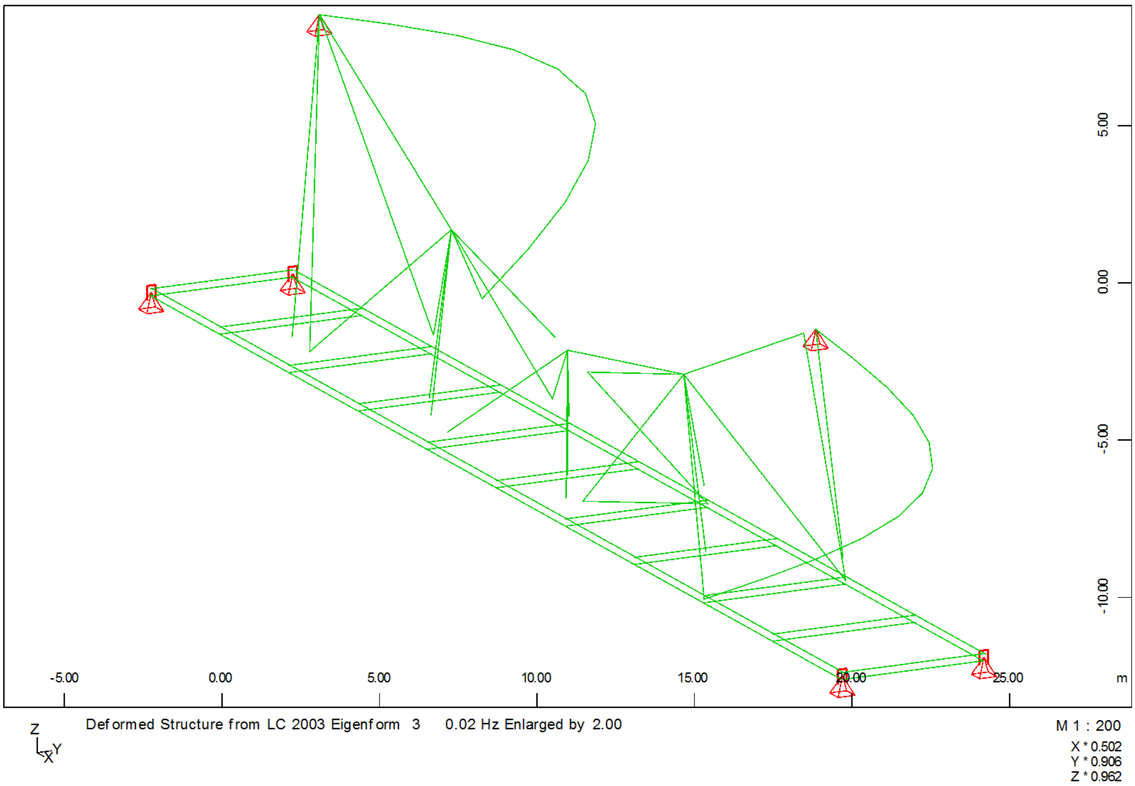

Fig. 3.24. 3rd eigenform

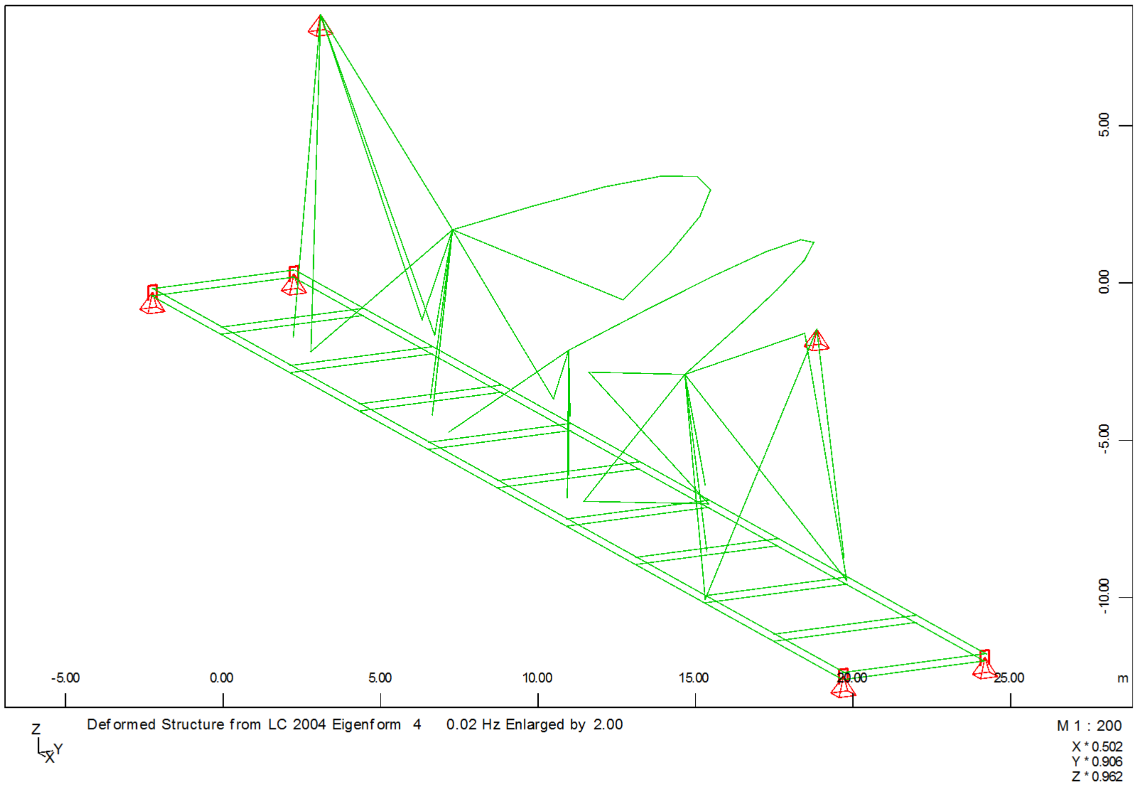

Fig. 3.25. 4th eigenform

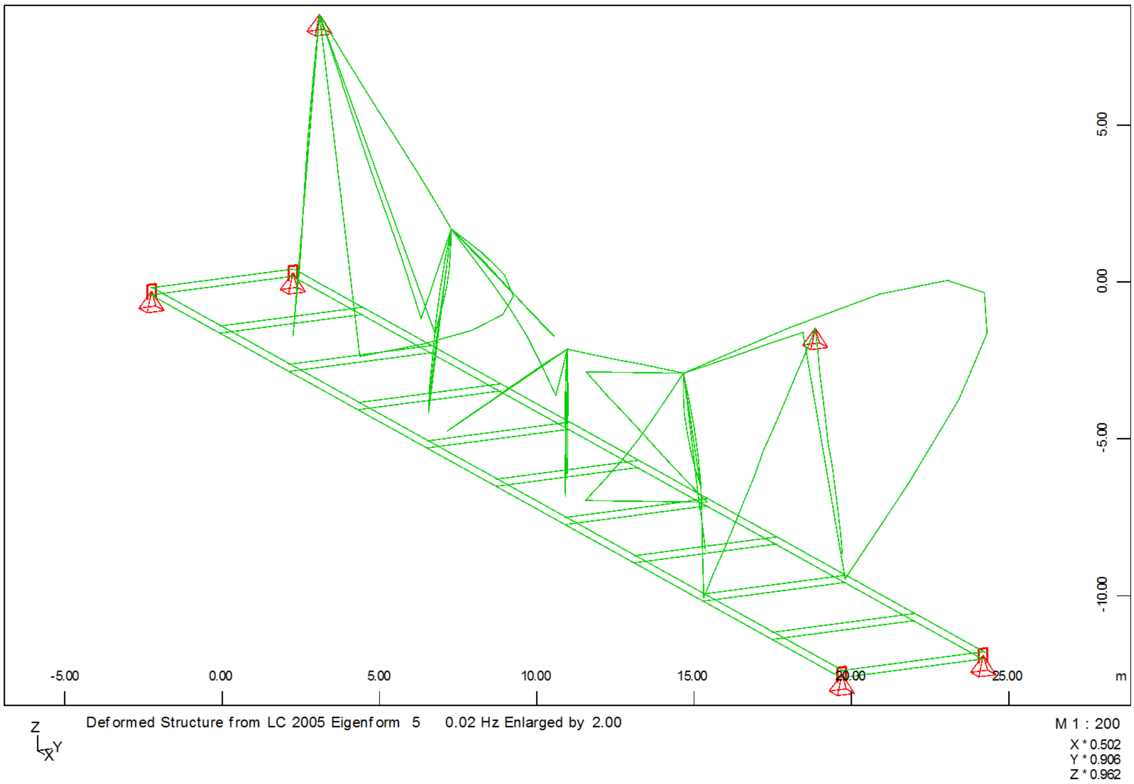

Fig. 3.26. 5th eigenform

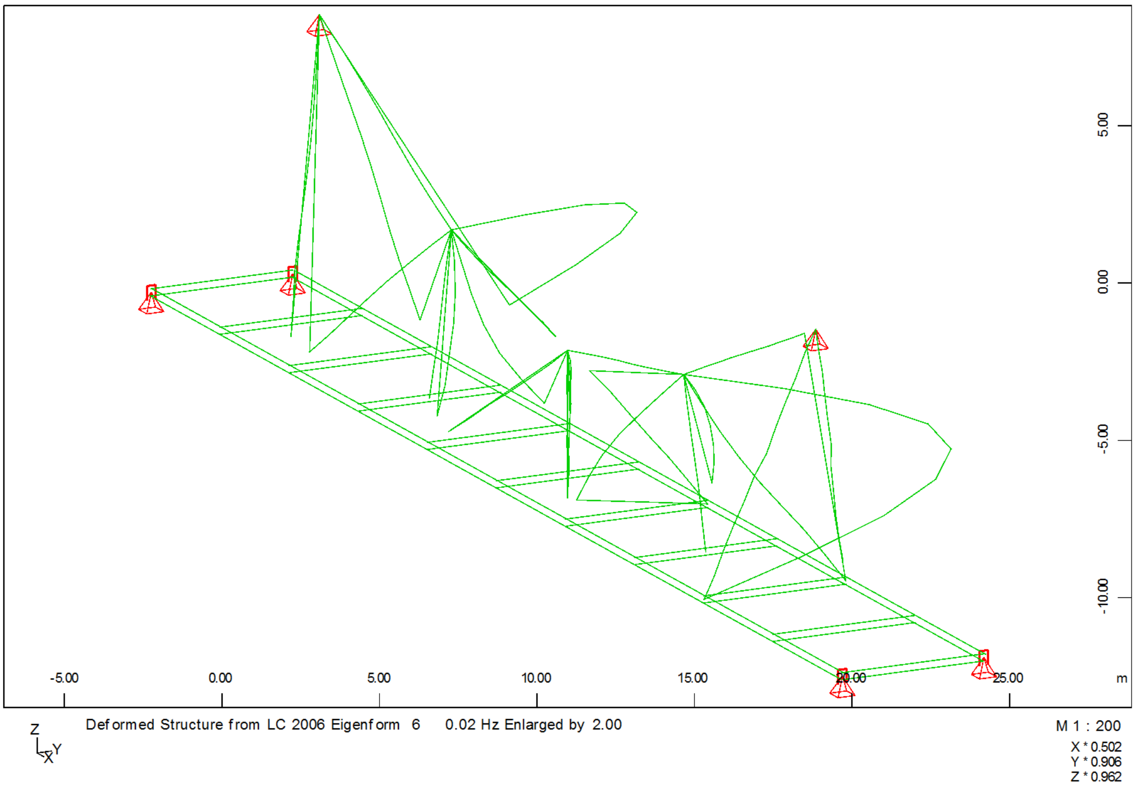

Fig. 3.27. 6[th] eigenform

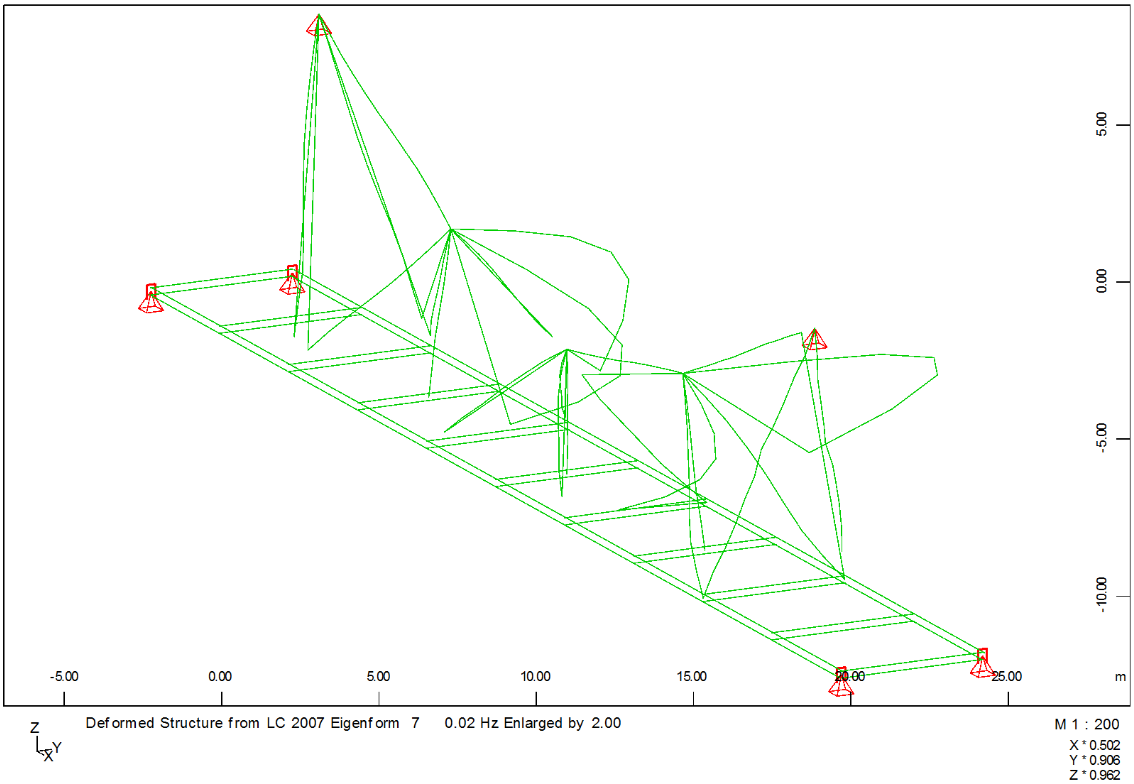

Fig. 3.28. 7[th] eigenform

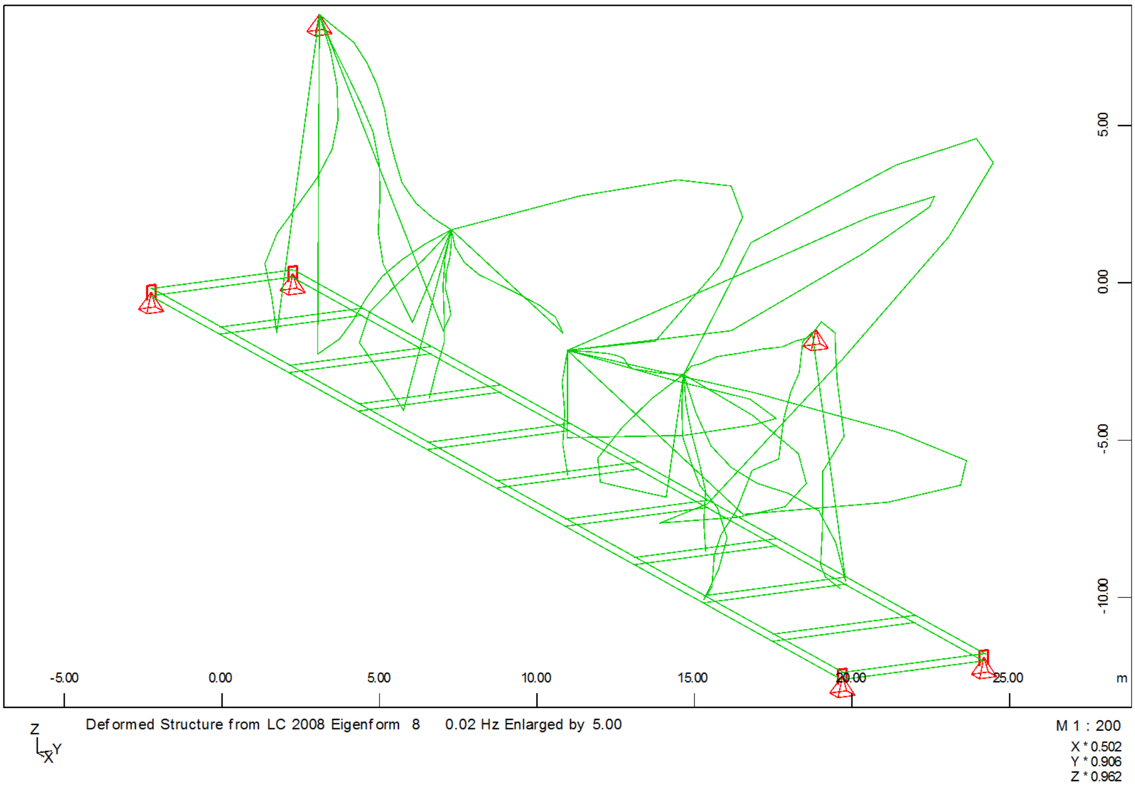

Fig. 3.29. 8th eigenform

Table 3.13. Natural frequencies of the main structure of the bridge deck

No.	Eigenvalue, $1/s^2$	Relative error	Frequency, *Hz*	Period, *s*
1	8.40261E-03	5.13E-06	0.015	68.544518
2	8.54655E-03	1.29E-05	0.015	67.964874
3	9.06294E-03	8.09E-06	0.015	66.000221
4	1.30939E-02	5.82E-04	0.018	54.909164
5	1.54399E-02	1.62E-02	0.020	50.565952
6	1.63515E-02	2.97E-02	0.020	49.136173
7	1.97408E-02	4.41E-02	0.022	44.719582
8	2.33209E-02	8.35E-02	0.024	41.144100

According to the results none of eight calculated frequencies get in critical range of natural frequencies.

After FEA of the tensegrity bridge it can be stated that bridge is stable and results obtained from form-finding process is reliable and can be used for further analysis of the structure.

3.5. Concluding Remarks of Chapter 3

Chapter 3 draws following key findings:

1. A conversion of EFDM appropriate to deal with tensegrity systems supporting beams in flexure has been presented. For a given assembly of tensile and compressive elements and loading condition, suggested improvement enables to find the particular prestressed system which replaces both static and kinematic functions of the inner reactions of an elastic flexural continuous beam.
2. Considered application of the tensegrity for the cable-stayed bridge shows the effectiveness and the adaptability of the suggested approach seeking to solve

problems of complex geometries and/or constrains and contributes to the design of new creative forms.

3. The FEA of the tensegrity bridge showed that data obtained from form-finding procedure through suggested algorithm of EFDM is reliable and can be used for further analysis of the structure.
4. Deference between maximum and minimum bending moments of the deck structure is 6.1%, that shows good distribution of bending moments and that pretension forces obtained from form-finding procedure are sufficient.
5. The tensegrity bridge structure is rigid, deflection of deck structure from SLS load combination is very small it is only 9.05% of limited maximal deflection.
6. Tensegrity bridge structure is stable, none of eight calculated frequencies get in critical range of natural frequencies.

GENERAL CONCLUSIONS

Present study suggests a possibility of applying tensegrity as constructive load carrying system in cable-stayed bridges and concludes as following:

1. An overview of cable-stayed bridges and tensegrity systems was presented. The idea of a cable-stayed bridge is straightforward. The deck supports the loads and stays provide medial supports for the bridge so that it can cross a long distance. By replacing usual cable-stays system with tensegrity system more technically and economically rational and architecturally expressive structures can be achieved.
2. Tensegrity structures integrate axially loaded discrete members without any bending moments in them. These structures are lightweight, adjustable and resourceful in mass.
3. When considering mass consumption tensegrity systems provide better capability to resist external loads in comparison to conventional structures. This property of tensegrity structures may possibly alternate to distinctive rigid structures. The tensegrity concept presents new opportunities for structural expression in civil engineering.
4. Tensegrity structures experience a highly efficient structural behaviour reasoned by exceptionally axial resistance, structural control performed by pretension of tensile members and easer predictable systematic instability. The extreme flexibility of tensegrity structures makes them capable to withstand large structural shocks and determines applicability for structures built at the areas of natural hazards. Due to wide variation in structural configuration tensegrity structures serve as a desirable tool for architects in order to satisfy strict aesthetic requirements and realize their self-expression; none the less they are desirable by structural engineers as a problem of form-finding.
5. A study of the existing form-finding methods for tensegrity structures and their categorization is done to set a right method for an outline of statically stable tensegrities. After analysis of existing form-finding methods for tensegrity structures, the FDM is chosen for its flexibility.
6. Adding constraints to FDM allows to set relative distances between the nodes, the tensile intensity in the elements and their initial length. To establish the positions of a particular number of nodes and, together, to impose the intensity of the reaction force the EFDM was proposed.
7. New arrangement of cable-staying system for cable-stayed bridge was proposed and the form-finding process through EFDM was performed. This process lets to determine tensegrity structure form by setting the topology of structure, loading conditions and type of each member. Considered application of the tensegrity for the cable-stayed bridge shows the effectiveness and the adaptability of the suggested approach seeking to solve problems of complex geometries and/or constrains and contributes to the design of new creative forms.
8. EFDM suitable to deal with tensegrity systems supporting beams and allows to find that particular pretensioning system which replaces both the static and the kinematic functions of the inner reactions of a flexural elastic continuous beam.
9. The FEA of the tensegrity bridge showed that data obtained from form-finding procedure through suggested algorithm of EFDM is reliable. Bridge structure is rigid and stable.

Suggestions for Future Research

Tensegrity systems use in large structures is limited due to the current lack of analytical tools to study and design tensegrity structures. In the future perspective, the following issues may become the subjects of studies:

1. To provide more accurate computational methods for the analysis of geometrically nonlinear behavior of tensegrity based cable-stayed bridge the following aspects could be taken into account: the beam-column effect, the large displacements effect and the cable sag effect.
2. By examining the relation among methods and how they solve the initial equilibrium problem the new hybrid approaches can be developed by combining strengths of these methods.
3. More detailed analysis of the tensegrity based cable-stayed bridge behaviour for asymmetric load should be done.

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APPENDICES

APPENDIX A

THE USE OF TENSEGRITY SYSTEMS IN SEARCH FOR NEW FORMS OF CABLE-STAYED BRIDGES: LITERATURE REVIEW

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Abstract. Tensegrity systems are technical structures consisting of tension and compression elements which are light weight, deployable, energy efficient, and highly controllable. The use of tensegrity systems for cable-stayed bridges, may prove to be very economical. The objective of this review paper is to understand the basic principles on which a tensegrity system is based. This paper reviews: a finite element computation procedure for the form-finding analysis of cable-stayed bridge and introduces the theoretical foundation of the proposed methods; definitions given by various researchers in the field of tensegrity; characteristics, advantages and disadvantages of these structures; various applications of tensegrity systems; form-finding methods of tensegrity systems.

Keywords: tensegrity systems, form-finding, cable-stayed bridge.

1. Introduction

Over the last half century, a large number of cable-stayed bridges has been built or are under construction all over the world (Wang *et al.* 1993; Wang, Yang 1995; Wang *et al.* 2003). Because of its aesthetic appeal, economic grounds and the ease of erection, the cable-stayed bridge is considered as most suitable for medium to long span bridges (Wang *et al.* 1993; Wang, Yang 1995; Wang *et al.* 2003). Use of tensegrity systems for this bridges, may prove to be very economical.

Tensegrity systems are technical structures consisting of tension and compression elements which are light weight, deployable, energy efficient, and highly controllable (Joshi, Al-Hakkak 2015; Rhode-Barbarigos *et al.* 2010; Ali *et al.* 2010; Nuhoglu, Korkmaz, K. 2011; Korkmaz, S. *et al.* 2011; Ali, Smith 2010; Juan, Tur 2007). A tensegrity structure is a paradigm of continual tension and intermittent compression. These structures essentially consist of bars and strings attached to the end of the bars, which are all loaded axially and do not receive bending moments (Joshi, Al-Hakkak 2015).

This paper reviews literature background of the current state of the art in form-finding for cable-stayed bridges and tensegrity structures. The paper is organized as follows: Section 2 reviews a finite element computation procedure for the form-finding analysis of cable-stayed bridge and introduces the theoretical foundation of the proposed methods. Section 3 presents the definitions given by various researchers in the field of tensegrity. Section 4 discus characteristics, advantages and disadvantages of these structures. Section 5 introduce various applications of tensegrity systems. Section 6 reviews form-finding methods of tensegrity systems. Finally, Section 7 concludes the paper.

2. Form-finding analysis of cable-stayed bridges

2.1. Finite element model

Based on the finite element theory, a cable-stayed bridge can be take into consideration as an assembly of a finite number of cable and beam (girder) column (tower) elements. In presented finite element computation approach for form-finding of cable-stayed bridges some assumptions are made as follows (Wang *et al.* 1993; Wang, Yang 1995; Wang *et al.* 2003). The material is homogeneous and isotropic and behaves linearly elastically. The external loads are displacement independent. All cables are fixed to the tower and to the girder at their joints of attachment. Large displacements and large rotations are allowed, but strain is small. The geometric nonlinearities are taken into account in the computation approach for form-finding of cable-stayed bridges.

2.2. Nonlinearities

There are three main sources of geometrically nonlinear behaviour of cable-stayed bridges which must be taken into account: the beam-column effect, the large displacements effect and the cable sag effect (Freire *et al.* 2006; Wang *et al.* 1993; Wang, Yang 1995; Wang *et al.* 2003).

The elastic cable is assumed to be perfectly flexible and have only tension stiffness; it is lacking ability of withstanding compression, shear and bending forces. The inclined cable stay of cable-stayed bridges is in general quite long and cable hold up at its end and under the action of dead load and axial tensile force will sag into catenary shape. Changing sag, the axial stiffness of a cable will change. When an unswerving cable element for a full-length inclined cable stay is used in the analysis, the sag effect has to be taken into account (Freire *et al.* 2006; Wang *et al.* 1993; Wang, Yang 1995; Wang *et al.* 2003).

The towers and part of the girders are subjected to a large compression action since a high pretension force exists in inclined cable stays; this means that the beam-column effect has to be taken into account for girders and towers of the cable-stayed bridge (Freire *et al.* 2006; Wang *et al.* 1993; Wang, Yang 1995; Wang *et al.* 2003).

Generally, cable-stayed bridges have an of considerable size span and smaller weight than that of standard steel and reinforced concrete bridges. Large deflections may easily emerge in cable-stayed bridges, this means that the large displacement effect has to be taken into account in the analysis and the equilibrium equations have to be set up based on the deformed position (Freire *et al.* 2006; Wang *et al.* 1993; Wang, Yang 1995; Wang *et al.* 2003).

2.3. General system equation

The general system equation for finite element model, derived from the virtual work principle have the following form (Wang *et al.* 1993; Wang, Yang 1995; Wang *et al.* 2003):

$$\boldsymbol{K}^j \cdot \boldsymbol{b}_\alpha^j - \sum_{EL} S_j a_{j\alpha} = 0 \quad \alpha = 1,2,\dots,N\ DOF, \qquad (1)$$

where $P_\alpha = \boldsymbol{K}^j \cdot \boldsymbol{b}_\alpha^j$ the generalized external forces, $\boldsymbol{K}^j$the external nodal load vectors, $\boldsymbol{b}_\alpha^j = \partial \boldsymbol{W}^j / \partial q_\alpha$ the basis vector, $\boldsymbol{W}^j$ the displacement vectors corresponding to $\boldsymbol{K}^j$, q_α the generalized systems coordinates, $T_\alpha = \sum_{EL} S_j a_{j\alpha}$ the generalized internal forces, $\sum_{EL}$ the summation over all elements, $S_j = \boldsymbol{KE}_{jk} u_j + S_j^0$ the generalized element forces, $\boldsymbol{KE}_{jk}$ the element stiffness matrix, u_j the generalized element coordinates, S_j^0 the generalized initial element forces, $a_{j\alpha} = \partial u_j / \partial q_\alpha$ the transformation coefficients, N the number of degree of freedom (DOF). The superscript j denotes the nodal number. The subscripts $\alpha, \beta, \gamma, \dots$ denote the number of the system coordinate and $j, k, l, \dots$ are the number of the element coordinate. The index summation convention is used here for the superscripts and subscripts. Letters printed in bold-face type, e.g. $\boldsymbol{K}^j$, $\boldsymbol{b}_\alpha^j$, represent vectors. The dot notation between vectors means scalar product. General system equation can be solved by the load increment methods or the iteration methods (Wang *et al.* 1993; Wang, Yang 1995; Wang *et al.* 2003).

2.4. Linearized system equation

In nonlinear statics the linearized finite element system equation, derived from the virtual work principle have the following form (Wang *et al.* 1993; Wang, Yang 1995; Wang *et al.* 2003):

$$\Delta P_\alpha^n + {}_u P_\alpha^n = {}^2K_{\alpha\beta}^n \cdot \Delta q_\beta^n \quad for\ P_\alpha^n \le P_\alpha \le P_\alpha^{n+1}, \qquad (2)$$

where $\Delta P_\alpha^n = P_\alpha^{n+1} - P_\alpha^n$ the load increments, ${}_u P_\alpha^n = P_\alpha^n - \sum_{EL} S_j^n a_{j\alpha}^n$ the unbalanced forces at nth load step in statics, ${}^2K_{\alpha\beta}^n = \sum_{EL} \boldsymbol{KE}_{jk}^n a_{j\alpha}^n a_{k\beta}^n + \sum_{EL} S_j^n a_{j\alpha,\beta}^n - {}^n\boldsymbol{K}^j \cdot {}^n\boldsymbol{b}_{\alpha,\beta}^j - {}^n\boldsymbol{K}_\beta^j \cdot {}^n\boldsymbol{b}_\alpha^j$ the tangent system stiffness matrix, $\Delta q_\alpha^n = q_\alpha^{n+1} - q_\alpha^n$ the displacement increments, $a_{j\alpha,\beta} = \partial a_{j\alpha} / \partial q_\beta$ the transformation coefficients of second-order, $\boldsymbol{b}_{\alpha,\beta}^j = \partial \boldsymbol{b}_\alpha^j / \partial q_\beta$. The superscript "$n$" denotes the number of load step, e.g., $q_\beta^n = q_\beta|_{P_\alpha = P_\alpha^n}$, and the number "2" means iteration matrix of second-order (Wang *et al.* 1993; Wang, Yang 1995; Wang *et al.* 2003).

2.5. Form-finding procedure

The form-finding of cable-stayed bridge gives the geometric shape as well as the prestress distribution of the bridge under the action of dead load of girders and towers and the pretension force in inclined cable stays. The analysis can be performed in two different ways: linear theory and nonlinear theory. Theoretically, exact and more smooth shape of the bridge girder can be found only by the nonlinear theory (Wang *et al.* 1993; Wang, Yang 1995; Wang *et al.* 2003).

Wang *et.al* (1993) and Wang, Yang (1995) present a finite element computation approach for form-finding of cable-stayed bridges under the action of the dead loads of girders and pretension in inclined cables, later Wang *et.al* (2003) set up a finite element computation approach for the form-finding analysis of cable-stayed bridges during erection procedures: forward process analysis and backward process analysis by using both the linear computation procedure and the nonlinear computation procedures. The former solved using the Newton-Raphson method and the latter solved using the cantilever method.

The Newton-Raphson method of form-finding analysis of cable-stayed bridges is in short outlined in the following (Wang *et al.* 1993; Wang, Yang 1995; Wang *et al.* 2003):

1) Input the geometric and physical data of the bridge.
2) Input the dead load of girders and towers.
3) Input the reference configuration (designed shape) of the bridge.
4) Input suitably estimated initial forces in cable stays to start the computation.
5) Find equilibrium position
   i. Linear procedure

- Linear cable and beam-column elements and linear constant coordinate transformation coefficients $a_{j\alpha}$ are used.
- Set up and solve the linear system equation for q_α (equilibrium position).
- No equilibrium iteration is carried out.

ii. Nonlinear procedure

- Nonlinear cable element with sag effect and beam-column element and nonlinear coordinate transformation coefficients $a_{j\alpha}$, $a_{j\alpha,\beta}$ are used.
- Set up and solve the incremental linearized system equation for Δq_α.
- Equilibrium iteration is performed by using the Newton–Raphson method.

6) Shape iteration

- Check if the convergence tolerance $\left|\frac{q_\alpha}{L_s}\right| \leq \varepsilon_s$ is achieved or not.
- If converged, the equilibrium configuration is the desired initial shape. Otherwise, the newly obtained axial forces of members are taken as initial element forces, repeat steps 5 and 6.

7) Output of the initial shape including geometric configuration and element forces.

The form-finding during erections procedures in detail analysed by Wang *et al.* (2003) The forward process analysis of cable-stayed bridges during construction is performed by following the sequence of erection stages in bridge construction, while the backward process analysis follows the reverse direction of the sequence of the bridge erection procedure. At each erection stage, the finite element analysis model is rebuilt, then the system equation is set up and solved anew under the action of dead load and member forces determined in the previous stage for finding the corresponding new initial shape (Wang *et al.* 2003).

3. Concept of tensegrity systems

Tensegrity systems are technical structures consisting of tension and compression elements which are light weight, deployable, energy efficient, and highly controllable (Joshi, Al-Hakkak 2015; Rhode-Barbarigos *et al.* 2010; Ali *et al.* 2010; Nuhoglu, Korkmaz, K. 2011; Korkmaz, S. *et al.* 2011; Ali, Smith 2010; Juan, Tur 2007). The word tensegrity is a combination of tensile and integrity, it was proposed by Richard Buckminster Fuller in 1962 (Joshi, Al-Hakkak 2015; Rhode-Barbarigos *et al.* 2010; Pagitz, Tur 2009; Amouri *et al.* 2014; Juan, Tur 2007). He described tensegrity systems as "*islands of compression in an ocean of tension"* (Joshi, Al-Hakkak 2015; Rhode-Barbarigos *et al.* 2010; Juan, Tur 2007), later in 1975 he defines tensegrity as follows:" *The word tensegrity is an invention: it is a contraction of tensional integrity. Tensegrity describes a structural-relationship principle in which structural shape is guaranteed by the finitely closed, comprehensively continuous, tensional behaviours of the system and not by the discontinuous and exclusively local compressional member behaviours. Tensegrity provides the ability to yield increasingly without ultimately breaking or coming asunder. The integrity of the whole structure is invested in the finitely closed, tensional-embracement network, and the compressions are local islands*." (Joshi, Al-Hakkak 2015).

Throughout the same decade, David George Emmerich in 1963 and Kenneth Snelson in 1965 patented similar systems (Joshi, Al-Hakkak 2015; Rhode-Barbarigos *et al.* 2010; Amouri *et al.* 2014; Juan, Tur 2007). Kenneth Snelson defines tensegrity system as follows: "*Tensegrity describes a closed structural system composed of a set of three or more elongate compression struts within a network of tension tendons, the combined parts mutually supportive in such a way that the struts do not touch one another, but press outwardly against nodal points in the tension network to form a firm, triangulated, prestressed, tension and compression unit.*" (Joshi, Al-Hakkak 2015). David George Emmerich describe tensegrity system as follows: "*Self-stressing structures consist of bars and cables assembled in such a way that the bars remain isolated in a continuum of cables. All these elements must be spaced rigidly and at the same time interlocked by the pre-stressing resulting from the internal stressing of cables without the need for external bearings and anchorage. The whole is maintained firmly like a self-supporting structure, whence the term self-stressing*." (Juan, Tur 2007).

Anthony Pugh in 1976 proposed definition, which is the result of combining the definitions by Richard Buckminster Fuller, David George Emmerich and Kenneth Snelson: "*A tensegrity system is established when a set of discontinuous compression components interacts with a set of continuous tensile components to define a stable volume in space.*" (Juan, Tur 2007).

A current and widely conceptual meaning was suggested by Rene Motro in 2003: "*A tensegrity is a system in stable self-equilibrated state comprising a discontinuous set of compressed components inside a continuum of tensioned components.*" (Joshi, Al-Hakkak 2015; Rhode-Barbarigos *et al.* 2010; Ali *et al.* 2010; Quirant *et al.* 2003; Ali, Smith 2010; Amouri *et al.* 2014; Juan, Tur 2007). This definition takes in systems where compressed elements are interconnected as tensegrity structures (Rhode-Barbarigos *et al.* 2010).

Another widely cited definition of a tensegrity systems is proposed by Robert Skelton in 2001: "*A Class k tensegrity structure is a stable equilibrium of axially loaded elements, with a maximum of k compressive members connected at the node(s).*" (Joshi, Al-Hakkak 2015; Juan, Tur 2007).

Tensegrity is a comparatively new area of study, currently none of the definitions have been invariably accepted, why it is very important to know the different definitions in a variety of disciplines and the differences among them (Joshi, Al-Hakkak 2015).

4. Characteristics of tensegrity structures

The main characteristics of tensegrity are presented as follows (Joshi, Al-Hakkak 2015; Skelton *et al.* 2001):

- Tensegrity structures are stable structures. They are also completely independent of external forces.
- In a tensegrity structure, members are designed for axial loads and none of the member experience bending moments. For this reason, more dependable and precise models can be presumed. Moreover, buckling load in compressive elements cannot be predicted exactly in application as equate the tensile strength of a member. For this reason, increased use of tensile members in tensegrity structures can help to have more precise models of the structures.
- Tensegrity structures are mass efficient. In tensegrity structures, longitudinal members are arranged in uncommon systems which are not orthogonal to achieve maximum strength with minimum mass. For this reason, bridges, domes etc. constructed using the principles of tensegrity may prove to be very economical.
- Tensegrity structures are deployable structures. Because the compressive members of tensegrity structures are either disjoint or connected with ball joints, large displacement, deployability, and stowage in a compact volume will be immediate virtues of tensegrity structures. This feature offers operational and portability advantages.
- Tensegrity structures vibrate readily and transfer loads rapidly. For this reason, none of the individual members experience any stresses locally. By the virtue of this property, tensegrity structures may prove useful in absorption of seismic shocks.
- In contrast to most of the orthodox structures and mechanisms, tensegrity structures can be controlled with much smaller control energy.
- Pre-tensioning grows the stiffness of tensegrity structures. But, to achieve high levels of stiffness, immense pre-tension in the strings is needed which may not be easy to execute. This is considered as the foremost disadvantage of these structures.
- The manufacture of tensegrity structures is not easy. Complicated structures like spherical and domical structures may be confronted by problems in fabrication.
- These structures are nonlinear and for that reason their modeling and control needs special methods which are still under progress.

5. Applications of tensegrity systems

Tensegrity structures find applications in a sort of fields like civil engineering, architecture, mechanical engineering, aerospace, and biomechanics (Joshi, Al-Hakkak 2015).

These days, material costs have been a point of concern and as material costs go on rising, tensegrity structures which use material more effective are becoming more and more attractive. Configuration of tensegrity structures is controllable. In case of earthquakes or wind loads adjustable tensegrity structures can be used to lower damage.

In domical arrangement, tensegrity structures can be used for manufacture of remarkably extensive structures, for diverse purposes such as extensive electrical or electromagnetic shielding, green house for plant and food production, etc. Tensegrity structures are deployable. For that reason, they can be used for disaster-stricken areas, building field hospitals, temporary shelters, etc.

Tensegrity systems can be used to build light weight shells for pavilions. In situations where large displacements are not a matter of concern or considerable displacements are acceptable, tensegrity towers can be employed to support antennas, receptors, radio transmitters, mobile telephone transmitters, etc. (Joshi, Al-Hakkak 2015).

With a lot of research works conducted by civil engineers and architect all over the world, it is now feasible to mathematically model and design tensegrity structures for architectural applications (Joshi, Al-Hakkak 2015). For this reason, tensegrity bridges (Fig. 1) and roofs are now a reality.

Fig. 81. Kurilpa bridge in Brisbane Australia built in 2009

6. Form finding methods of tensegrity systems

Form-finding methods for tensegrity structures have been analysed by many authors. Existing form-finding methods for tensegrity structures can be classified into two categories: kinematical methods and statical methods (Tibert, Pellegrino 2011; Pagitz, Tur 2009; Juan, Tur 2008). The former methods are outlined by increasing (decreasing) the length of the struts (cables) and keeping the length of the cables (struts) constant until a maximum (minimum) is reached (Tibert, Pellegrino 2011; Pagitz, Tur 2009; Juan, Tur 2008). In these methods it is not needed that the members be in a state of pre-stress. In latter methods, when topology and the forces in its members are given, relationship is set up between equilibrium configurations of a structure (Tibert, Pellegrino 2011; Pagitz, Tur 2009; Juan, Tur 2008). This link is then analysed by different techniques.

In kinematical methods category, Connelly and Terrell (1995) used an analytical method, where the coordinates of each node are expressed as a function of geometric parameters and then maximize (minimize) the length expressions for struts (cables) given the length of the cables (struts) starting from an arbitrary configuration (Tibert, Pellegrino 2011; Pagitz, Tur 2009; Juan, Tur 2008). This method is simple for very symmetric structures, but it is infeasible for non-symmetric tensegrities due to the large number of variables needed (Juan, Tur 2008). Research in this field also have been done by: Koohestan, Guest (2013); Li *et al.* (2010).

Pellegrino (1986) and Burkhardt (2006) proposed non-linear programming technique by turning the form-finding method of tensegrity structures into another one of constrained minimization problem. For these methods need a correct configuration to start with and then try to minimize (maximize) the length of same struts (cables), but they do not take into account any stress restriction. For that reason, in spite of the fact that geometrically correct, the outcome structure may not be firm (Juan, Tur 2008). Research in this field also have been done by Burkhardt (2006).

Finally, Motro (1984) and Belkacem (1987) suggested the dynamic relaxation method for tensegrity structures. In order to get the equilibrium configuration, this method solves a fictitious dynamic model in terms of the acceleration, velocity and displacement from the initial configuration like the one shown below:

$$\boldsymbol{M}\ddot{\boldsymbol{d}} + \boldsymbol{D}\dot{\boldsymbol{d}} + \boldsymbol{K}\boldsymbol{d} = \boldsymbol{f}, \tag{3}$$

where $\boldsymbol{K}$ is a stiffness matrix, $\boldsymbol{M}$ a mass matrix, $\boldsymbol{D}$ a damping matrix, $\boldsymbol{f}$ the vector of external forces, and $\ddot{\boldsymbol{d}}$, $\dot{\boldsymbol{d}}$ and $\boldsymbol{d}$ the vectors of acceleration, velocity and displacement from the initial configuration, respectively (Juan, Tur 2008; Tibert, Pellegrino 2011). This method is only practical for small size structures, but it takes into account equilibrium considerations and the existence of external forces (Juan, Tur 2008). Research in this field also have been done by: Ali *et al.* (2010); Zhang *et al.* (2006).

In statical methods category Kenner (1976) applied node equilibrium conditions and symmetry arguments to come up with the stable configurations of some simple tensegrity structures (Juan, Tur 2008). This method is alike to Connelly and Terrell (1995) method for kinematic analytical approach but it guarantees stability of the structure without any external load (Juan, Tur 2008).

Schek (1974) and Linkwitz (1999) suggested the force density method which transforms the nonlinear equilibrium equations into a set of linear equations (Juan, Tur 2008; Tibert, Pellegrino 2011). This method demands the priory understanding of the stress coefficients for all members which is one of its greatest disadvantage since some combinations of stresses may not have physical applications in a given space (Juan, Tur 2008). Another disadvantage of this method is that it is not possible to control the length of the members (Juan, Tur 2008). The equilibrium equation of a general pin-jointed structure is

$$\boldsymbol{C}^T\boldsymbol{Q}\boldsymbol{C}\boldsymbol{p}_i + \boldsymbol{C}^T\boldsymbol{Q}\boldsymbol{C}_f\boldsymbol{p}_{i,f} = \boldsymbol{f}_i, \tag{4}$$

where $\boldsymbol{C}$ and $\boldsymbol{C}_f$ is the incidence matrixes for a given topology, $\boldsymbol{Q}$ is a diagonal matrix containing the force density coefficients, $\boldsymbol{p}_i, \boldsymbol{p}_{i,f}$ and $\boldsymbol{f}_i$ are the coordinate vector and the external force applied to each node in the ith direction, respectively. The product $\boldsymbol{C}^T\boldsymbol{Q}\boldsymbol{C}$ ($\boldsymbol{C}^T\boldsymbol{Q}\boldsymbol{C}_f$) is called the force density matrix (Juan, Tur 2008; Tibert, Pellegrino 2011). Research in this field also have been done by: Aboul-Nasr, Mourad (2015); Miki, Kawaguchi (2010); Quagliaroli, Malerba (2013); Tran, Lee (2009); Tran, Lee (2010a); Tran, Lee (2010b); Tran, Lee (2010c); Xu, Luo (2009) Zhang, Ohsaki (2006).

Energy based form-finding method was presented by Connelly (1993). In this method he assigns an energy function to a tensegrity and searches the minimum of this function, which is equivalent to test the positive semi-definiteness of the stress matrix ($\boldsymbol{\Omega}$) (Juan, Tur 2008; Tibert, Pellegrino 2011). This stress matrix is identical to the force density matrix used in the force density method (Juan, Tur 2008; Tibert, Pellegrino 2011).

Another method was introduced by Sultan *et al.* (1999). Consider a set of generalized coordinates for a particular tensegrity framework and use symbolic manipulation to obtain the equilibrium matrix ($\boldsymbol{R}(\boldsymbol{p})$) (Juan, Tur 2008; Tibert, Pellegrino 2011). Sultan (1999) specified the pre-stress state for a two stage SVD tensegrity structure which can be specified with just three parameters (i.e. the azimuth (α), the declination (δ) and the overlap (h)) (Juan, Tur 2008; Tibert, Pellegrino 2011). Research in this field also have been done by Tur, Juan (2008).

Micheletti and Williams (2004) introduce a method based on solving a system of differential equations. These authors also suggest a method to modify a given stable configuration to get to one more by changing the length of a given edge and solving the system of differential equations to get the change in length of the other edges (Juan, Tur 2008).

Zhang *et al*. (2006) developed a method to first find a set of axial forces compatible with a given structure and then find the relative nodal coordinates under equilibrium conditions and structure constraints (Juan, Tur 2008). This method mainly obtains the bases for the self-stress and positioning subspaces and then requires to fix a number of stresses and coordinates equal to the dimension of those subspaces, respectively, in order to find the final solution (Juan, Tur 2008).

Masic *et al*. (2005) introduced a modified version of the force density method introduced by Schek (1974) and Linkwitz (1999) which directly involves form constraints. They also studied how the symmetry properties can be used to systematically reduce the number of force density variables, equilibrium equations and geometrical variables (Juan, Tur 2008). They put on view that the equilibrium of a tensegrity structure is invariant under an affine transformation (i.e.$\overline{\boldsymbol{p}} = \boldsymbol{Sp} + \boldsymbol{r}$) of the nodal coordinates (Juan, Tur 2008).

Paul *et al*. (2005) presented a different approach based on genetic algorithms to find the topology which assures stability. In which genetic algorithms are used to evaluate an initial arbitrary topology into a stable one in the work space (Juan, Tur 2008). This method is fit to generate asymmetrical structures. Research in this field also have been done by: Chisari *et al.* (2015); Gan *et al.* (2015); Yamamoto *et al.* 2011; Koohestani (2012); Skelton *et al.* (2013).

Masic *et al*. (2006) using non-linear programming techniques introduced by Pellegrino (1986), developed a method which search for the topology, geometry and pre-stress of a structure under external forces, and taking into account strength, buckling and form constraints (Juan, Tur 2008). The application of sequential quadratic programming (a penalty method) allows the algorithm to find stable configurations starting from a random one, but some uniqueness's in the gradient function due to the use of the element length may source the algorithm to diverge or as an alternative, link up to a non-optimum solution (Juan, Tur 2008). Also, their form-finding method holds some physical phenomena due to external loads (Juan, Tur 2008).

Finally, Estrada *et al*. (2006) introduced a method which only requires information about the type of each edge and about the topology, but does not account for external forces (Juan, Tur 2008). Then, the equilibrium geometry and force densities for each edge are iteratively calculated using rank constraints on both the stress matrix ($\boldsymbol{\Omega}$) and the rigidity matrix ($\boldsymbol{R}(\boldsymbol{p})$) (Juan, Tur 2008). Research in this field also have been done by: Cheong *et al.* (2014); Koohestani, Guest (2013).

In Table 1 there is a summary of the form-finding methods (Juan, Tur 2008).

Table 14. Summary of form-finding methods of tensegrity systems

Method name	Class	Assures stability	Needs a valid initial configuration	Uses symmetry	Needs an initial topology	Uses external forces
Analytic solution	Kinematic	No	No	Yes	Yes	No
Non-linear programming	Kinematic	No	Yes	No	Yes	No
Dynamic relaxation	Kinematic	Yes	Yes	No	Yes	Yes
Analytic solution	Static	Yes	No	Yes	Yes	No
Force density method	Static	Must be given	No	No	Yes	Yes
Energy method	Static	Yes	No	No	Yes	No
Reduced coordinates	Static	Yes	No	Yes	Yes	No
Differential equations	Static	Yes	Yes	No	Yes	No
Successive approximation	Both	Some stresses have to be fixed	Some coordinates have to be fixed	No	Yes	No
Algebraic method	Static	Must be given	No	No	Yes	Yes
Genetic algorithm	Topologic	Yes	No	No	No	No
Sequential quadratic programming	Both	Yes	No	No	No	Yes
Numerical Method	Both	Yes	No	No	Yes	No

7. Conclusions

This review paper is focused on understanding the definitions, characteristics, applications and form-finding methods of tensegrity structures, also a finite element computation procedure for form-finding analysis of cable-stayed bridges is in short presented.

A tensegrity system is a particular case of a truss in which structural members have particular functions. All the members in a tensegrity structure are axially loaded and not one of the single members undergo bending moment. Tensegrity structures provide better ability to withstand external loads as compared to typical rigid structures with comparable mass. These structures are lightweight, deployable and mass efficient. In tensegrity structures, longitudinal members are arranged in uncommon systems which are not orthogonal, maximum mass efficiency is achieved having maximum strength with minimal mass. This property of tensegrity structures makes them a possible alternative for typical rigid structures. The tensegrity concept presents new opportunities for structural expression in civil engineering.

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