ISSN 1648-4142 print / ISSN 1648-3480 online TRANSPORT
www.transport.vgtu.lt

TRANSPORT – 2006, Vol XXI, No 4, 274–277

EQUIVALENT MECHANICAL PARAMETERS OF OSCILLATING ROTARY MOTORS USED IN TRANSPORT EQUIPMENT

Arūnas Andziulis[1], Jonas Vaupšas[1], Eleonora Guseinovienė[1], Rimantas Didžiokas[2], Danielius Adomaitis[2]

[1]*Faculty of Marine Engineering, Klaipėda University, Bijūnų g. 17, LT-91225 Klaipėda, Lithuania. Phone +370 686 21 147. E-mail: arunas@ik.ku.lt*
[2]*Mechatronics Institute, Klaipėda University, Bijūnų g. 17, LT-91225 Klaipėda, Lithuania.*

Received 5 June 2006; accepted 4 September 2006

Abstract. In various appliances and equipment of sundry transport means there is a lot of diverse mechanisms of periodical movement. So, the various piston or membrane pumps of fuel feeding and lubrication systems, circulation pumps, air and refrigerating coolant compressors, etc. are the typical examples of innovative and well promising application of the oscillating motors. In these cases the moving part of a motor can be directly connected to the working body of driven mechanism without the additional gears. Consequently, the drive can be simplified in design and improved in efficiency and reliability. Application of the oscillating rotary motors, if used in the aforesaid devices, strictly depends on specific properties of mechanical system of a motor aggregated with the driven mechanism and considered as the one-piece unit on the whole. So, this study analyses how the properties of mechanical system, comprised of two moving parts interconnected eccentrically or centrically, can be evaluated by the equivalent rotational inertia, equivalent mass and by equivalent mechanical power factor which, in turn, determine the operating characteristics and basic possibilities of the motor.

Keywords: transport equipment, oscillating rotary motors, mechanical system, equivalent parameters.

1. Introduction

Owing to specific temporal and spatial properties of mechanical movement the oscillating rotary motors are widely used in drives of mechanisms equipped with the working body of rotary and linear periodical motion and used in various arrangements of modern transport equipment. Identity between the main parameters of mechanical movement of motor and driven device (for example, accordance in frequency and amplitude) gives the possibility of immediate coupling of the motor's moving part and operational unit of the driven mechanism.

This present-day tendency enables to simplify a mechanical system of the drive and to avoid additional reduction gears or converters of mechanical motion. In turn, it ensures the higher energy efficiency, reliability and lasting working qualities of the driven device. There is a wide range of numerous mechanisms which can be considered as the advanced and typical cases of successful application of the oscillating motors of linear or rotary movement. So, the various types of piston and membrane pumps of fuel supply and lubrication systems, circulation and delivery low-lift pumps, air and refrigerating coolant compressors, diverse units of reciprocating devices of various purposes are the characteristic drives of the oscillating movement [1–3].

In accordance with the modern theory of oscillating electrical machines, the electrical, magnetical and the mechanical systems can be relatively discerned in a structure of the motor of any type [4, 5]. Many parameters and variables of these systems are tightly interrelated between themselves, which, in turn, causes the strict predetermination of main abilities and features of the oscillating drive on the whole. However, in particular the parameters of mechanical system usually exert the substantial influence on the main operating characteristics of a motor.

The analysis discloses that the basic properties of mechanical system, which consists of two rotary oscillating parts, can be evaluated by equivalent rotary (angular) inertia and equivalent mechanical power factor, etc. So, the definition of equivalent mechanical parameters is the important objective which allows facilitating problems of more precious study and practical designing of the oscillating rotary motors.

2. Load and the type of mechanical system of a motor

In typical steady-state operational mode the main variables of a motor are practically harmonic or contain some highly expressed sinusoidal components of appropriate frequencies. Therefore, in accordance with analogy of differential equations, which similarly describe the oscillatory processes which are going on in electrical or mechanical system, the load of an oscillating motor can be expressed by mechanical impedance, i.e. by the complex mechanical resistance *Z:*

$$Z = R + jX = R + \left(X_J - X_S\right) = R + j\left(\omega J_e - \frac{1}{\omega s}\right), \quad (1)$$

here: R , X_J, X_s – active mechanical resistance, inertial reactance and resilient reactance of a spring; ω, J_e , s – cyclic frequency, equivalent rotary inertia, rotational compliance of a spring.

So, the load and accordingly the operational characteristics and performance factors of an oscillating rotary motor are definitely depending on the active mechanical resistance resulted from driven device as well as on the mechanical reactance, which, in turn, is specified by masses, rotary (angular) inertia of moving parts and resiliency (torsion rigidity) of the compensating spring. Consequently, owing to periodicity of mechanical movement, the equivalent rotary inertia of mechanical system plays the essential part in all operating values of an oscillating rotary motor.

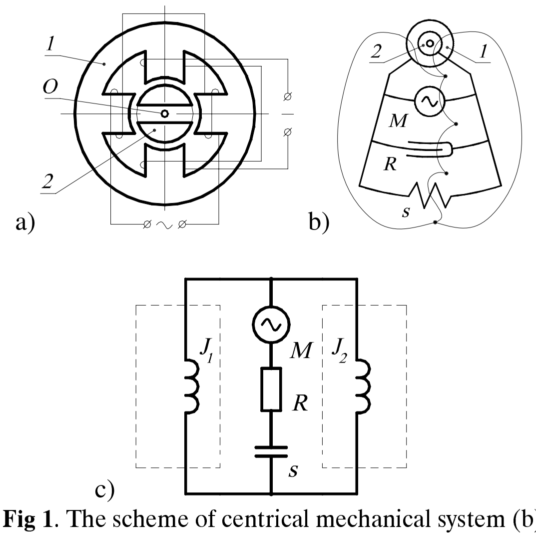

Fig 1. The scheme of centrical mechanical system (b) of an oscillating rotary motor (a) and its electrical analogue (c): 1– stator; 2 – rotor; O – point of connection

Unlike conventional motors of permanent rotary movement the oscillating motors are usually designed according to the specified requirements and particularity of the driven device. It causes the wide variety of types and constructions of the oscillating motors. However, the mechanical system of an ordinary oscillating motor, distinguishing by one degree of freedom (Fig 1 a and Fig 2 a), typically consists of two parts 1 and 2, which can be only relatively considered as the traditional *stator* and *rotor*.

Both parts, pivotally connected between them and interacted by alternating electromagnetic torque, practically are oscillating at different amplitudes according to higher or lesser values of their rotary inertia. So, the equivalent parameters of two-mass oscillating mechanical system should be defined by taking into account masses and rotary inertia of both parts of the motor.

In spite of the great diversity of types and constructions, the mechanical systems of oscillating motors can be divided into two groups with respect to positional relationship of mass-centres of the moving parts. The so-called centrical mechanical system embodies the motors in which the mass-centres of both moving parts are ordered on the one axis passed through the common point of their connection (hinge or bearing). However, the eccentrical mechanical system is typical for much wider group of motors, in which the mass-centres of each moving part (or at least of one of them) are positioned eccentrically with respect to the axis drawn through the hinge. The schemes of the oscillating motors of centrical and eccentrical mechanical system are given in Fig 1 a and in Fig 2 a respectively.

The inertness of each moving part of centrical system can be directly expressed by the rotary inertia defined with respect to the axis that is drawn through the common mass-centre of parts and the hinge perpendicularly to the plain of oscillation. In case of eccentrical mechanical system the moving parts rotationally oscillate about the designated instantaneous speed centres [6], which are definitely positioned on the plain of oscillation dependently on ratios of masses, rotary inertia and on the geometry of mutual displacement of hinge and mass-centres of the parts. In this case, the inertness of moving parts can be evaluated by only the equivalent rotary inertia calculated in accordance with the mentioned peculiarities of eccentrical system. However, in case of centrical or eccentrical system the inertial properties of a motor as well as the inertial reactance of a load can be appropriately expressed by the equivalent parameters which should be taken into account in solving problems of analysis or synthesis of these motors.

3. Equivalent parameters of centrical and eccentrical mechanical system

The evaluation of inertness of mechanical system of a motor on the whole can be facilitated by working out the schemes-analogues of mechanical system which can be composed by following theory and method of the dual circuits [7]. So, in accordance with the constructional type of a motor there can be composed the corresponding mechanical circuit of the mechanical system [4, 8].

The mechanical circuit of centrical system (in traditional mechanical symbols) is presented in Fig 1 b, in which some closed contours can be traced for composing of dual topological graph. In this case the nodal points of dual circuit can be designated inside of each independent contour of the mechanical circuit while the one “dependent point” is to be fixed outside borders of the circuit. In accordance with the basics of dual circuits the lines connecting all the nodal points (thin lines in Fig 1 b) should be drawn through each element of mechanical circuit.

So, the electrical scheme – analogue of mechanical system of a motor can be composed by sequential following the configuration and structure of the graph and in accordance with the principles of the chosen analogy “force-voltage, speed-current”.

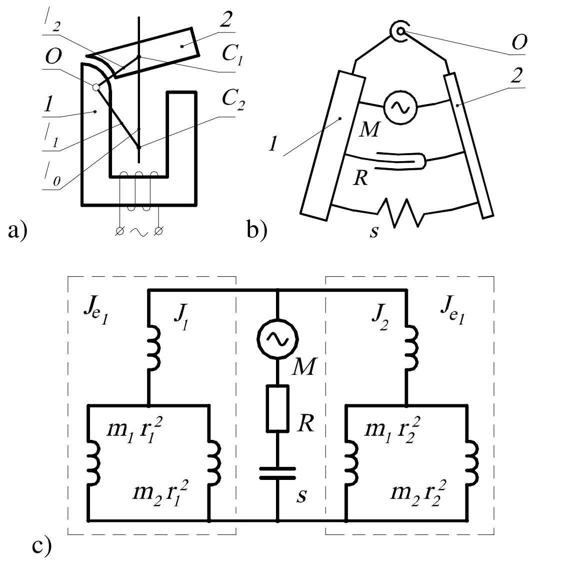

Fig 2. The scheme of eccentrical mechanical system (b) of an oscillating rotary motor (a) and the electrical analogue with the detailed parallel branches (c)

The electrical analogue of centrical mechanical system is presented in Fig1 c in which the electrical capacitance corresponds to rotational compliance s of the spring and the inductances of parallel branches represent the rotational inertia J_{e1} and J_{e2} of moving parts defined in respect to axis passed through their mass-centres perpendicularly to the plain of oscillation.

Now, the equivalent rotary inertia J_e of mechanical system of two-mass motor on the whole (Fig 3) can be easily defined by taking into account the structure of electrical analogue and by adapting the expression of resulting resistance of parallel branches:

$$J_e = \frac{J_1 \cdot J_2}{J_1 + J_2}\,. \qquad (2)$$

In fact, the equation (2) can be applied for any type of mechanical system of a motor. However, the calculation of equivalent rotary inertia of eccentrically connected parts is much more complicated. In this case, the equivalent inertia can be defined in respect to the axes passed through the aforesaid instantaneous speed centres [6] that, in turn, needs for detailed elaboration of parallel branches of electrical analogue despite the apparent similarity of dual graph of mechanical circuit.

This problem can be worked out by joint solution of equations of angular momentum conservation law which, in turn, can be compiled under assumption of positional invariability of total mass-centre of mechanical system and under the fulfilment restriction of small amplitudes of oscillation.

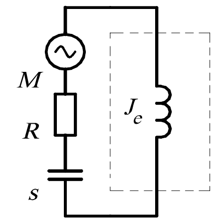

Fig 3. The generalized electrical analogue of mechanical system of the oscillating motor

The usage of additional equations, which describes the geometry of displacement of hinge and mass-centres of the parts, enables to work out details of parallel branches of electrical analogue. So, the advanced electrical analogue of eccentrical mechanical system of motor is presented in Fig 2 c in which m_1 , m_2 and r_1 , r_2 are the masses of moving parts and radiuses of inertia, which, in turn, depend on distance l_0 between the mass-centres C_1 and C_2 (Fig 2 a) and on the distances l_1 and l_2 of centres from the hinge. So, the equivalent rotary inertia J_{e1} and J_{e2} of moving parts of a motor, calculated in respect of the electrical analogue of mechanical system, can be expressed by formulas (3) and (4) in which m_e is the equivalent (reduced) mass of stator and rotor respectively:

$$J_{e1} = J_1 + m_e r_1^2 = J_1 + \frac{m_1 \cdot m_2}{m_1 + m_2} \cdot \frac{l_0^2 + l_1^2 - l_2^2}{2}\,, \qquad (3)$$

$$J_{e2} = J_2 + m_e r_2^2 = J_2 + \frac{m_1 \cdot m_2}{m_1 + m_2} \cdot \frac{l_0^2 - l_1^2 + l_2^2}{2}\,. \qquad (4)$$

If one of the two parts distinguishes by far greatly inertia (for example, if the stator of motor is connected with massive mainframe of the driven device, i.e. in case of $J_{e1} \gg J_{e2}$), then the structure of electrical analogue can be simplified and composed of the only one parallel branch which corresponds to inertia of rotor. So, in accordance with (2) the equivalent rotary inertia of mechanical system becomes equal to rotary inertia of the part of lesser inertia, i.e. $J_e = J_{e2}$. In this case, the equation of equivalent rotary inertia J_{e2} can be reduced to the simplified version which is well-known as the formula of parallel-axis theorem

$$J_{e2} = J_2 + m_2 l_2^2 . \qquad (5)$$

The methods used in typical calculation of electrical circuitry, as applied to detailed form of electrical analogue (Fig 2 c), enable to derive the generalized equation of equivalent rotary inertia J_e of eccentrical mechanical system of the motor (Fig 3) –

$$J_e = \frac{J_1 J_2 + m_e\left(J_1 r_2^2 + J_2 r_1^2 + m_e r_1^2 r_2^2\right)}{J_1 + J_2 + m_e\left(r_1^2 + r_2^2\right)} . \qquad (6)$$

Consequently, equations of equivalent rotary inertia can find use in corresponding calculation of some basic operating factors or constructional parameters of a motor. So, the following application of mentioned analogy and usage of equation (6) allows to calculate inertial reactance of complex mechanical resistance of load (1) and to define the equivalent mechanical power factor $\cos\psi$:

$$\cos\psi = \frac{R}{\sqrt{R^2 + \left(\omega J_e - \frac{1}{\omega s}\right)^2}} . \qquad (7)$$

In turn, the use of equation (2) and (7) gives a possibility of further calculation of torsion rigidity k of spring of mechanical system:

$$k = \omega\left(\omega J_e - R \cdot tg\psi\right) \qquad (8)$$

as well as to define the consumption of mechanical power P_m in the active resistance R of load:

$$P_m = \frac{1}{2} R\omega^2 \theta_2^2 \left(1 + \frac{J_{e2}}{J_{e1}}\right)^2 , \qquad (9)$$

where θ_2 is an angular amplitude of oscillation of the rotor.

For various constructional types of motor the rotary inertia J_1 and J_2 (with respect to axes passed through mass-centres etc.) can be defined by experimental measurement of frequency of free oscillation while the active resistance of load can be calculated by applying the expression of logarithmic decrement defined from examination of damped oscillation:

$$R = \frac{kT}{2\pi^2} \ln \frac{\theta_2(t)}{\theta_2(t+T)} , \qquad (10)$$

where $\theta_2(t)$, $\theta_2(t+T)$ are the angular amplitudes at an instant t and after the period T.

So, the results of study presented in this work can give certain information about the principal properties of mechanical system and can be used for more detailed and quantitative analysis or synthesis of the oscillating rotary motors and thereby can stimulate and widen fields of successful application of these motors.

4. Conclusions

There are many various devices and mechanisms of periodical movement that are wide spread and often used in the present-day transport means and that can be considered as the advanced and well promising field of application of the oscillating motors. The analysis of specific features of oscillating rotary motors, consisting of two moving parts (stator and rotor), interconnected eccentrically or centrically, enables to draw the following conclusions:

1. The inertial properties of mechanical system of an oscillating rotary motor consisting of stator and rotor as well as the operating mode of such motor can be quantitatively evaluated by the equivalent rotary inertia and equivalent power factor and by the mechanical resistance of load.
2. Equations of equivalent parameters can be derived by usage of electrical analogues composed according to the configuration and structure of topological dual graph of mechanical system of a motor.
3. The values of equivalent parameters are strictly depending on masses and rotary inertia of stator and rotor as well as on the geometry of displacement of hinge with respect to mutual allocation of mass-centers of these parts.

References

1. Wang, Y.; Megli, T.; Haghgooie, M.; Peterson, K. S.; Stefanopuolu, A. G. Modeling and control of electromagnetical valve actuator. *Society of Automotive Engineers,* SAE, Paper No. 2002-01-1106, 2002.
2. Kim, J. W.; Heo, J. T.; Kim, J. D. Linear compressor for natural refrigerant. In: Transactions of Int. Conference on Compressors and their Systems, IMechE, London, 2003, p. 483–491.
3. Ulbrich, H. Comparison of different actuator concepts for applications in rotating machinery. *International Journal of Rotating Machinery*, Vol 1, 1, 1994, p. 61–71.
4. Kudarauskas, S. Introduction to oscillating electrical machines. Klaipėda: Klaipėda University Press, 2004. 184 p.
5. Choe, G. S.; Kim, K. J. Theoretical and experimental analysis of nonlinear dynamics in a linear compressor. *Journal of Vibration and Acoustics,* Vol 124, 2002, p. 152–154.
6. Kudarauskas, S.; Vaupšas, J. Rotary vibration reduced at the speed centres. In: Proc. of Intern. Seminar on Vibration and Acoustic Noise of Electrical Machinery, Béthune, France 1999, p. 177–178.
7. Zeveke, G.; Ionkin, P.; Netushil, A.; Strakhov, S. Analysis of electric circuits. Moscow: Mir Publishers, 1979 (in Russian).
8. Lenk, A. Elektromechanische systeme. Berlin: VEB Verlag Technik, 1971.